Package Sizing and Pricing in an Emerging Market

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Abstract

Emerging markets like China and India are becoming dominant in the global economy. Such markets have a majority of poor, cash-constrained consumers, living on less than $3 a day. This paper studies the implications of global firms selling high quality products in relatively smaller package sizes in such emerging markets.

We develop a model to compare the package sizing and pricing decisions of two competing high- and low- quality firms in an emerging market, with a benchmark, developed market. In the model, the emerging market has two segments of consumers - a cash-constrained segment, which can afford the low quality product, but cannot afford the high quality product and an unconstrained segment which can afford both products. In contrast, the benchmark, developed market has only unconstrained consumers. In the emerging market, the firms' decision-making is in three stages. First, the high-quality firm decides to either exclusively sell to the unconstrained segment, or alternately, lower its price and sell to both segments. Second, both firms set their package sizes and finally, their prices.

The purpose of the analysis is to compare the firm's pricing and package sizing decisions in the presence of cash-constrained consumers, with their decisions in the absence of cash-constrained consumers. The analysis suggests that in the case when the high-quality firm lowers its price and sells to both segments in the emerging market, it also reduces its package size. More interestingly, the low-quality firm leaves its package size unchanged and raises its price in the emerging market, relative to the developed market. In the second case, when the high-quality firm exclusively sells to the unconstrained segment in the emerging market, each firm leaves its package size unchanged and raises its price, relative to the developed market.

The main contribution of this paper is to show that low-quality products sell for higher prices in the presence of cash-constrained consumers in emerging markets, as compared to their prices in a benchmark market without cash-constrained consumers. Importantly, this is true, regardless of whether the high quality firm chooses to exclusively sell to unconstrained consumers or chooses to sell to unconstrained and cash-constrained consumers by setting a smaller package size. Moreover, in both cases, consumer surplus is lower and the low-quality firm earns a higher profit. These results have important managerial implications for package sizing and pricing decisions in emerging markets.

Keywords: International Marketing, Package Sizing, Emerging Markets, Game Theory, Competitive Strategy

JEL: D11, D40, F00, M31

1. Introduction

1.1 Background

Emerging markets such as India and China are characterized by a sizable population of low-income consumers with limited ability to pay for consumer packaged goods. More than 4 billion people worldwide live on less than $3000 per year. Despite such cash constraints, these consumers collectively represent significant purchasing power (Prahalad 2006).

Global manufacturers frequently adopt a strategy of selling products in relatively smaller package sizes in emerging markets. For example, Colgate MaxFresh toothpaste is typically sold in 40 g, 80 g, and 150 g tubes in India while it is sold in 6 oz. (170 g) and 8 oz. (227 g) tubes at Target stores in the United States. Research from A.C. Nielson, India also shows that in the shampoo category, 50 ml and 100 ml sized bottles accounted for 73% of the annual 2007 revenue from shampoo sales. This reflects the heterogeneity in package size choices made by firms across emerging and developed markets. The objective of this article is to analyze the strategic implications of firms selling products in smaller package sizes in emerging markets.

Global firms such as Proctor & Gamble and Unilever, frequently compete with local firms selling lower quality alternatives in emerging markets. For example, in the detergent category in India, Proctor & Gamble's *Tide* detergent faces stiff competition from *Nirma* – a lower quality, cheaper detergent manufactured by a local Indian firm. This article focuses on how consumers' limited ability to pay in emerging markets affects firm strategy in a competitive setting. Specifically, when a high quality global firm sets a small package size in order to sell to cash-constrained customers in an emerging market, how should this influence the pricing and package sizing decision of a local firm selling a lower quality competing product?

The presence of cash-strapped consumers in emerging markets directly affects pricing strategy. If a firm charges a price higher than consumers' cash constraint, irrespective of their willingness to pay, their limited ability to pay precludes sales. The presence of cash-constrained consumers however, also indirectly affects other strategic variables such as package sizing. The Marketing literature, with a few exceptions (e.g., Gerstner and Hess 1987; Koenigsberg et al. 2008), provides insufficient insight into firms' package sizing decision; this research is a step in this direction.

Emerging markets are becoming dominant in the global economy; for example, the Morgan Stanley MSCI Emerging Markets Index, which measures the equity market performance of 25 emerging markets, has increased by 300% in the past five years. Brazil, Russia, India, and China, known as BRIC, collectively account for 15% of the global economy (Goldman and Sachs 2007). India maintains more than 12 million retail outlets, with estimated annual sales of $215 billion in 2005. Moreover, the total retail market in India is likely to grow at a 5.5% compounded annual growth rate, to $374 billion, by 2015 (Tata Strategic Management Group 2006). Despite these staggering numbers, conducting business in emerging markets poses some unique strategic challenges and decisions for global manufacturers such as Unilever, Procter and Gamble. Furthermore, the existing marketing literature, with a few notable exceptions (e.g., Chandrasekaran and Tellis 2008; Hofstede et al 2002), has not studied emerging markets. This paper attempts to fill this void.

1.2 Model Overview and Key Results

A high- and low-quality firm each sells a single product in a horizontally and vertically differentiated market. Each firm sets the package size and price for its product. Product quality is exogenous and each firm's marginal cost is proportional to its package size. Consumer utility increases linearly with product quality and concavely with package size. The market contains two segments, with consumers uniformly distributed along a Hotelling line. A market segment is either *unconstrained*, which indicates that consumers can afford to purchase both products, or it is *cash-constrained*, in which case consumers can afford to purchase only the cheaper product.

A benchmark, developed market has two unconstrained segments. In contrast, an emerging market has one unconstrained and a second cash-constrained segment. In the emerging market, the firms' decisions are in three stages. First, the high-quality firm either decides to sell to both market segments by lowering its price to match the limited ability to pay of the cash-constrained segment or alternately it exclusively sells to the unconstrained segment. In the second and third stages, both firms set their package sizes and prices respectively. The analysis compares the package sizing and pricing decisions in the emerging market, relative to the developed market.

When the high quality firm sells to both segments in the emerging market at the cash constrained price, it correspondingly decreases its package size relative to the developed market. In contrast, the low quality firm maintains the same package size and interestingly, sets a relatively higher price in the emerging market.

When the high quality firm exclusively sells to the unconstrained segment in the emerging market, it maintains the same package size as in the developed market. The low quality firm also leaves its package size unchanged in both markets. However, both firms set higher prices in the emerging market, as compared to the developed market.

In the emerging market, the profit of the high quality firm from selling to both segments exceeds its profit from exclusively selling to the unconstrained segment when consumers' price sensitivity is relatively low and vice-versa.

The low quality firm makes a higher profit in the emerging market relative to the developed market, regardless of whether the high quality firm sells to one or two market segments. Also, the resulting consumer surplus in the emerging market is relatively less than that in the developed market in both situations.

The remainder of this article proceeds as follows: Section 2 positions the paper relative to prior related literature. Section 3 develops a benchmark model for a developed market and also a model of an emerging market. It compares the emerging market with the developed market. Finally, Section 4 features the conclusion and discusses some limitations of this research.

2. Related Literature

Virtually no prior research investigates package sizing in emerging markets, with the notable exception of Koenigsberg et al. (2008), who concentrate on single-serve package sizes or sachets for impoverished consumers. They develop a model in which a firm sells a product with a finite usable life to consumers with heterogeneous usage rates and reservation quantities. They find that even if there are no constraints on the income streams of buyers, a seller can charge higher unit prices and sell a larger quantity of a product by making single-serve packages. They conclude that a single-serve package permits the seller to price discriminate among buyers with different usage requirements, such that each consumer self-selects to purchase quantities that match his or her desired level of consumption as closely as possible.

This paper does not focus on important aspects such as the heterogeneity in products' usable life, consumers' usage rates and reservation quantities in emerging markets, as studied by Koenigsberg et al. (2008). This paper complements the research contribution by Koenigsberg et al. (2008), by focusing on the strategic and competitive implications of consumers' limited ability to pay in emerging markets.

Surprisingly little research investigates the firm's package sizing problem, with the exception of Gerstner and Hess (1987), who consider the package sizing and pricing problem of a monopolist. They show that consumer heterogeneity in consumption rates, storage costs, and transaction costs explains differences in package sizes and unit prices, such that heterogeneity can lead to unit prices that reflect quantity discounts or quantity premiums. This article also considers package sizing but in the context of an emerging market that contains consumers with heterogeneous abilities to pay. That is, this investigation does not explicitly model consumer heterogeneity in consumption rates and storage costs but rather focuses on how the presence of a cash-constrained market segment with limited ability to pay influences firms' package sizing and pricing decisions.

A small literature stream pertains to issues other than package sizing in the context of emerging markets. For example, Chandrasekaran and Tellis (2008) study the global takeoff of new products across multiple countries and product categories. They find that the average time to takeoff differs across countries depending on their culture, wealth, and product class and varies substantially between developed countries and emerging markets. The present investigation complements such research into emerging markets by focusing on the firms' package sizing decision.

The Marketing literature has a long history of modeling product differentiation and price competition (e.g., Balasubramanian 1998; Chen and Iyer 2002; Desai 2001; Iyer and Pazgal 2003; Pazgal and Soberman (2008); Kuksov 2004; Lal and Sarvary (1999); Moorthy 1988; Shaffer and Zhang 1995; Tyagi 2004; Villas-Boas 2004; Xie and Shugan 2001), and the modeling approach herein is consistent with such literature.

3. Model

The market contains two competing firms $\{H, L\}$. Each firm sells a single product of quality $\{q_h, q_l\}$ and package size $\{s_h, s_l\}$ for price $\{p_h, p_l\}$, respectively. Product quality is exogenously determined, with $q_h > q_l > 0$. Each firm's decision variables refer to its package size and price. Each firm's marginal cost increases linearly with its package size, as $c_h = k_h s_h$ and $c_l = k_l s_l$, respectively, where $k_h > 0$ and $k_l > 0$ are positive constants.

The market contains two segments of relative sizes λ and $(1-\lambda)$. A market segment may be either *unconstrained* or *cash-constrained*; it is unconstrained if the consumers in the segment can afford to purchase either of the two products $\{H, L\}$. Analogously, a market segment is cash-constrained if its consumers can afford to purchase the cheaper product but not the more expensive product. If χ represents the maximum amount of cash available to consumers in a cash-constrained segment, then $(p_l < \chi < p_h)$. An emerging market has one unconstrained segment and a second cash-constrained segment. A benchmark, developed market has two unconstrained segments.

The benefit from purchasing a product depends jointly on its product quality and package size. The benefit from a product increases linearly with product quality q and concavely with package size s. Let $f(s)$ be a positive, increasing, concave function that represents the benefit from purchasing a product of package size s, where $f(s) > 0$, $f'(s) > 0$, and $f''(s) < 0$. The concavity of *f(s)* reflects the diminishing marginal returns from larger package sizes. Let θ denote consumers' marginal valuation for product quality and package size together. Therefore, consuming a product of quality q and size s provides a standalone benefit of $\theta q f(s)$.

Define the *base value* (β) of a product of quality q, package size s, and price p as $\beta = \theta q f(s) - p > 0$. The base values of the high- and low-quality products, respectively, are thus

$$\beta_h = \theta q_h f(s_h) - p_h > 0, \text{ and} \quad (1)$$

$$\beta_l = \theta q_l f(s_l) - p_l > 0. \quad (2)$$

The base values of the products are strictly positive; otherwise no consumer will make a purchase.

A firm's margin is the difference between its price and marginal cost, or $m = p - ks$. The margins for the high- and low-quality products are thus $m_h = p_h - k_h s_h > 0$ and $m_l = p_l - k_l s_l > 0$, respectively.

Define a product's *value* α as the sum of the *base value* and margin, or $\alpha = \beta + m$. The *value* of a product α gets divided between the consumers and the firm as the consumers' *base value* β and the firm's margin *m*. Since $\beta = \theta q f(s) - p > 0$ and $m = p - ks$, so $\alpha = \theta q f(s) - ks$, and the values of the high- and low-quality products, respectively, are

$$\alpha_h = \theta q_h f(s_h) - k_h s_h > 0 \text{, and} \quad (3)$$

$$\alpha_l = \theta q_l f(s_l) - k_l s_l > 0 . \quad (4)$$

In turn, $(\alpha_h > \beta_h > 0)$ and $(\alpha_l > \beta_l > 0)$; otherwise, the firms prefer to not sell their products, and consumers prefer not to make a purchase.

To capture consumers' heterogeneous preferences, a Hotelling model with horizontal differentiation is appropriate (Hotelling 1929). Consumers in each market segment are uniformly distributed across a unit interval $x \in (0,1)$, and the firms' location decision is exogenous. Without loss of generality, let the high-quality firm H be located at the left end $(x = 0)$ and the low-quality firm L be located at the right end $(x = 1)$ of the Hotelling line. The utility functions of the consumers in the market, which they derive from purchasing one unit of the high- or low-quality product, are $U_h(x) = \theta q_h f(s_h) - p_h - tx$ and $U_l(x) = \theta q_l f(s_l) - p_l - t(1-x)$, respectively. These utility functions may be rewritten in terms of the base values of H and L, respectively, as

$$U_h(x) = \beta_h - tx \text{, and} \quad (5)$$

$$U_l(x) = \beta_l - t(1-x) . \quad (6)$$

The *base value* of a product represents the maximum utility that a consumer can obtain from purchasing that product. A consumer located at $x = 0$ on the Hotelling line achieves a utility of $U_h(0) = \beta_h$ from purchasing the high-quality product. Similarly, a consumer located at $x = 1$ on the Hotelling line earns a utility of $U_l(1) = \beta_l$ from purchasing the high-quality product.

The unit transportation cost $(t > 0)$ indicates the importance (unimportance) of preference (price); it also may be interpreted as a measure of consumers' *price insensitivity* (Kim et al. 2001). A consumer located at x on the Hotelling line incurs a transportation cost tx to purchase the high-quality product H or a transportation cost of $t(1-x)$ to purchase the low-quality product L. These costs represent the mismatch between this consumer's ideal product and the products $\{H, L\}$. In turn, the utility of a consumer located at x on the Hotelling line, obtained from purchasing a product, equals the difference between the product's *base value* and the consumer's transportation cost.

This preliminary analysis is of a benchmark, developed market, in which both market segments are unconstrained. Subsequently, the analysis considers an emerging market in which one segment is cash constrained and the second segment is unconstrained. The high-quality product is assumed to be more expensive than the low-quality product. In the presence of a cash-constrained segment, the high-quality firm must either lower its price to match the limited ability to pay of the cash-constrained segment and thus sell to both segments or choose not to lower its price and sell exclusively to the unconstrained segment. The subsequent analyses consider both these alternatives in the emerging market.

3.1 A Benchmark, Developed Market

A benchmark developed market has two unconstrained market segments. Consider the first unconstrained segment of relative size λ. An unconstrained consumer located at x on the Hotelling line purchases the high-quality product H if $U_h(x) > U_l(x)$ and $U_h(x) > 0$. Alternately, the consumer purchases the low-quality product L if $U_l(x) > U_h(x)$ and $U_l(x) > 0$. Each consumer purchases a single unit of either H or L, and no consumer purchases both H and L. Let $x = x_0$ indicate the location of an unconstrained consumer indifferent between purchasing $\{H, L\}$, which implies that $U_h(x_0) = U_l(x_0)$, or $x_0 = \frac{\beta_h - \beta_l + t}{2t}$. Unconstrained consumers located between $(0 < x < x_0)$ on the Hotelling line purchase H; unconstrained consumers located between $(x_0 < x < 1)$ on the Hotelling line purchase L.

Let $x = x_{h0}$ indicate the location of the unconstrained consumer who is indifferent between purchasing H and not purchasing, and let $x = x_{l0}$ indicate the location of the unconstrained consumer who is indifferent between purchasing L and not purchasing. Solving $U_h(x_{h0}) = 0$, $U_l(x_{l0}) = 0$ yields $x_{h0} = \frac{\beta_h}{t}$ and $x_{l0} = 1 - \frac{\beta_l}{t}$. Consumers therefore achieve a positive utility from purchasing only if $(x_{l0} < x_0 < x_{h0})$.

Let $\{d_h, d_l\}$ denote the aggregate sales of $\{H, L\}$ across both the unconstrained segments. Since the relative sizes of the segments are λ and $(1-\lambda)$, $d_h = \lambda x_0 + (1-\lambda)x_0 = x_0$, and $d_l = \lambda(1 - x_0) + (1-\lambda)(1 - x_0) = (1 - x_0)$, respectively. In addition, the margins of $\{H, L\}$ are $m_h = p_h - k_h s_h$ and $m_l = p_l - k_l s_l$, respectively, so the profit functions of $\{H, L\}$ are $\Pi_h = m_h d_h$ and $\Pi_l = m_l d_l$, or

$$\Pi_h = \left(p_h - k_h s_h\right)\left(\frac{\beta_h - \beta_l + t}{2t}\right), \text{ and} \quad (7)$$

$$\Pi_l = \left(p_l - k_l s_l\right)\left(1 - \frac{\beta_h - \beta_l + t}{2t}\right). \quad (8)$$

In the benchmark model, when both market segments are unconstrained, the firms' decisions consist of two stages. In Stage 1, both firms $\{H, L\}$ simultaneously choose their profit-maximizing package sizes $\{s_h^*, s_l^*\}$. In Stage 2, both firms $\{H, L\}$ simultaneously choose their profit-maximizing prices $\{p_h^*, p_l^*\}$. Solving this game by backwards induction yields the firms' equilibrium package sizes and prices. The derivations are available in the appendices.

In Stage 1, the high- and low-quality firms set their equilibrium package sizes $\{s_h^*, s_l^*\}$, such that $\left(\theta q_h f'(s_h) - k_h\right) = 0$ and $\left(\theta q_l f'(s_l) - k_l\right) = 0$, or

$$f'(s_h^*) = \frac{k_h}{\theta q_h}, \text{ and} \quad (9)$$

$$f'(s_l^*) = \frac{k_l}{\theta q_l}. \quad (10)$$

In Stage 2, the firms set the following equilibrium prices:

$$p_h^* = \frac{3t + \alpha_h - \alpha_l}{3} + k_h s_h^*, \text{ and} \quad (11)$$

$$p_l^* = \frac{3t - \alpha_h + \alpha_l}{3} + k_l s_l^*, \quad (12)$$

where $\alpha_h = \theta q_h f(s_h) - k_h s_h$ and $\alpha_l = \theta q_l f(s_l) - k_l s_l$.

The equilibrium margins of the high- and low-quality firms are $m_h^* = \frac{3t + \left(\alpha_h - \alpha_l\right)}{3}$ and $m_l^* = \frac{3t - \left(\alpha_h - \alpha_l\right)}{3}$, and their equilibrium sales are $d_h^* = \frac{3t + \left(\alpha_h - \alpha_l\right)}{6t}$ and $d_l^* = \frac{3t - \left(\alpha_h - \alpha_l\right)}{6t}$, respectively. Note that the margins, sales, and profits of the high quality product exceed the low quality product, if the *value* of *H* is higher than *L* $\left(\alpha_h > \alpha_l\right)$. The resulting equilibrium profits of H and L are

$$\Pi_h^* = \frac{1}{2t}\left(t + \frac{\alpha_h - \alpha_l}{3}\right)^2, \text{ and} \quad (13)$$

$$\Pi_l^* = \frac{1}{2t}\left(t - \frac{\alpha_h - \alpha_l}{3}\right)^2, \tag{14}$$

where $\alpha_h = \theta q_h f(s_h) - k_h s_h$ and $\alpha_l = \theta q_l f(s_l) - k_l s_l$. The equilibrium base values of the high- and low-quality products are

$$\beta_h = \theta q_h f(s_h^*) - p_h^* = \frac{2\alpha_h + \alpha_l}{3} - t \text{, and} \tag{15}$$

$$\beta_l = \theta q_l f(s_l^*) - p_l^* = \frac{\alpha_h + 2\alpha_l}{3} - t\,. \tag{16}$$

Thus, the base value of the high-quality product exceeds the base value of the low-quality product$\left(\beta_h > \beta_l\right)$ if $\left(\alpha_h > \alpha_l\right)$.

To compare the equilibrium prices of the high- and low-quality products, equations (11) and (12) indicate that the difference in equilibrium prices is

$$p_h^* - p_l^* = \frac{2\left(\alpha_h - \alpha_l\right) + 3\left(k_h s_h^* - k_l s_l^*\right)}{3}\,. \tag{17}$$

The analysis focuses on when the high-quality product is relatively more expensive than the low-quality product. From (17), notice that $\left(\alpha_h > \alpha_l\right)$ and $\left(k_h s_h^* > k_l s_l^*\right)$ are sufficient conditions for the price of the high-quality product to exceed the price of the low quality product. In other words, the high-quality product is relatively more expensive if its *value* is higher than the low-quality product $\left(\alpha_h > \alpha_l\right)$ and if the marginal cost of producing the high-quality product is relatively greater than that of the low-quality product$\left(k_h s_h^* > k_l s_l^*\right)$. The remainder of this analysis assumes that these conditions are satisfied. Next, Lemma 1 compares the equilibrium package sizes of the high- and low-quality products in the benchmark developed market.

Lemma 1: *In the benchmark, developed market, the high-quality firm sets a smaller package size than the low-quality firm, i.e.* $s_h^* < s_l^*$, *if* $k_h > \left(\frac{q_h}{q_l}\right) k_l$.

Consider the impact of an increase in consumers' marginal valuation for product quality and package size jointly, given by θ. An increase in θ causes both firms to increase their equilibrium package size and price correspondingly, because $\frac{\partial s_h^*}{\partial \theta} > 0$, $\frac{\partial p_h^*}{\partial \theta} > 0$, $\frac{\partial s_l^*}{\partial \theta} > 0$, and $\frac{\partial p_l^*}{\partial \theta} > 0$.

Next consider the impact of an increase in consumers' price sensitivity. Recall that the unit transportation cost t is a measure of consumers' price insensitivity. In other words, a decrease in the unit transportation cost t correspondingly increases consumers' price sensitivity. The equilibrium package sizes set by the firms are independent of consumers' price sensitivity, because $\frac{\partial s_h^*}{\partial t} = 0$, and $\frac{\partial s_l^*}{\partial t} = 0$. An increase in consumers' price sensitivity correspondingly decreases the equilibrium prices, because $\frac{\partial p_h^*}{\partial t} = \frac{\partial p_l^*}{\partial t} > 0$.

Furthermore, the unit transportation cost t is bounded, as follows:

$$\frac{\alpha_h - \alpha_l}{3} < t < \frac{\alpha_h + \alpha_l}{3}. \qquad (18)$$

The lower bound is necessary to ensure that the margins are strictly positive, and the upper bound ensures that consumers obtain a strictly positive utility from their purchase. This concludes the analysis of a benchmark, developed market, where the consumers are unconstrained. The subsequent analysis considers an emerging market.

3.2 An Emerging Market

An emerging market has two consumer segments: an unconstrained segment of relative size λ and a cash-constrained segment of relative size $(1-\lambda)$. The cash-constrained segment can only afford to purchase the less expensive product. Since the high-quality product is relatively more expensive, $\left(p_l^* < \chi < p_h^*\right)$, where χ represents the cash-constrained segment's maximum ability to pay, and p_l^* and p_h^* are the equilibrium prices of the low- and high-quality products in the benchmark model. This segment finds the more expensive product to be unaffordable, so regardless of their valuation, consumers in this segment are unable to purchase a product priced higher than χ. In contrast, the unconstrained segment can afford to purchase either product.

The presence of the cash-constrained segment implies that the high-quality firm faces two alternatives in the emerging market. In the first alternative, it lowers its price to match the cash constraint χ and sells to both the unconstrained and cash-constrained segments. With the second alternative, it does not lower its price, exclusively sells to the unconstrained segment, and generates zero sales in the cash-constrained segment of the emerging market.

The firms' decision making now takes place in three stages. In Stage 1, the high-quality firm H decides between selling to both segments and selling exclusively to the unconstrained segment. In Stage 2, both the high- and low-quality firms $\{H, L\}$ simultaneously choose their profit-maximizing package sizes. In Stage 3, they simultaneously choose their profit-maximizing prices. Two possible subgames occur in Stages 2 and 3. Backwards induction provides the means to solve each subgame independently and thereby determine the firms' equilibrium package sizes and prices. The subscript *1* indicates when firm *H* sells to one segment, and the subscript *2* indicates when firm *H* sells to both segments in the emerging market.

3.2.1 High-quality Firm exclusively sells to the Unconstrained Segment in the Emerging Market

Let the price and package size set by the high-quality firm be p_{h1} and s_{h1}, respectively, when it exclusively sells to the unconstrained segment in the emerging market. Analogously, let the price and package size set by the low-quality firm be p_{l1} and s_{l1}. The prices are related, as $p_{l1} < \chi < p_{h1}$. The utility functions of consumers in the unconstrained segment are

$$U_{h1}(x) = \beta_{h1} - tx \text{, and} \tag{19}$$

$$U_{l1}(x) = \beta_{l1} - t(1-x) \text{,} \tag{20}$$

where β_{h1} and β_{l1} represent the base values of H and L, respectively, as follows:

$$\beta_{h1} = \theta q_h f(s_{h1}) - p_{h1} > 0 \text{, and} \tag{21}$$

$$\beta_{l1} = \theta q_l f(s_{l1}) - p_{l1} > 0 \text{.} \tag{22}$$

Since the cash-constrained consumers cannot afford to purchase the high-quality product, the utility functions are $U_{h1}(x) = 0$ and $U_{l1}(x) = \beta_{l1} - t(1-x)$.

First, consider the unconstrained segment of relative size λ. An unconstrained consumer located at x purchases the high-quality product H if $U_{h1}(x) > U_{l1}(x)$ and $U_{h1}(x) > 0$. Alternately, the consumer purchases the low-quality product L if $U_{l1}(x) > U_{h1}(x)$ and $U_{l1}(x) > 0$. Every unconstrained consumer in the market purchases a single unit of either H or L, and no unconstrained consumer purchases both H and L. Let $x = x_{u1}$ indicate the location of an unconstrained consumer indifferent between purchasing $\{H, L\}$. Then, $U_{h1}(x_{u1}) = U_{l1}(x_{u1})$, or $x_{u1} = \frac{\beta_{h1} - \beta_{l1} + t}{2t}$. Unconstrained consumers located between $0 < x < x_{u1}$ purchase the high-quality product H, whereas unconstrained consumers located between $x_{u1} < x < 1$ purchase the low-quality product L.

Second, consider the cash-constrained segment of relative size $(1-\lambda)$. No cash-constrained consumer purchases H because it is too expensive $(p_{h1} > \chi)$. A cash-constrained consumer located at x purchases L if $U_{l1}(x) > 0$ but does not purchase if $U_{l1}(x) < 0$. Let $x = x_{c1}$ indicate the location of a cash-constrained consumer indifferent between purchasing L and not making a purchase. Then, $U_{l1}(x_{c1}) = \beta_{l1} - t(1-x) = 0$, or $x_{c1} = \frac{t - \beta_{l1}}{t}$. Cash-constrained consumers located between $(0 < x < x_{c1})$ do not make a purchase, whereas those consumers located between $(x_{c1} < x < 1)$ purchase the low-quality product L.

Let $\{d_{h1}, d_{l1}\}$ denote the aggregate sales of $\{H, L\}$ across the unconstrained and cash-constrained segments. Since the relative sizes of these segments are λ and $(1-\lambda)$, respectively, $d_{h1} = \lambda x_{u1}$, and $d_{l1} = \lambda(1 - x_{u1}) + (1-\lambda)(1 - x_{c1}) = 1 - \lambda x_{u1} - (1-\lambda)x_{c1}$. Since the marginal costs of $\{H, L\}$ are, respectively, $c_{h1} = k_h s_{h1}$ and $c_{l1} = k_l s_{l1}$, the margins of $\{H, L\}$ are $m_{h1} = (p_{h1} - k_h s_{h1})$ and $m_{l1} = (p_{l1} - k_l s_{l1})$. Therefore, the profit functions of $\{H, L\}$ are $\Pi_{h1} = m_{h1} d_{h1}$ and $\Pi_{l1} = m_{l1} d_{l1}$, yielding

$$\Pi_{h1} = \lambda(p_{h1} - k_h s_{h1})\left(\frac{\beta_{h1} - \beta_{l1} + t}{2t}\right), \text{ and} \tag{23}$$

$$\Pi_{l1} = (p_{l1} - k_l s_{l1})\left(1 - \lambda\left(\frac{\beta_{h1} - \beta_{l1} + t}{2t}\right) - (1-\lambda)\left(\frac{t - \beta_{l1}}{t}\right)\right). \tag{24}$$

Analogous to the benchmark model, the *values* of the high- and low-quality products equal the sum of the *base values* and margins of the high- and low-quality products, $\alpha_{h1} = \beta_{h1} + m_{h1}$ and $\alpha_{l1} = \beta_{l1} + m_{l1}$, which yields

$$\alpha_{h1} = \theta q_h f(s_{h1}^*) - k_h s_{h1}^* > 0 \text{, and} \tag{25}$$

$$\alpha_{l1} = \theta q_l f(s_{l1}^*) - k_l s_{l1}^* > 0 . \tag{26}$$

In addition, $(\alpha_{h1} > \beta_{h1} > 0)$ and $(\alpha_{l1} > \beta_{l1} > 0)$; otherwise, the firms will not sell their products, and consumers will not make purchases.

In Stage 2, both firms $\{H, L\}$ simultaneously choose their profit-maximizing package sizes $\{s_{h1}^*, s_{l1}^*\}$, and in Stage 3, they simultaneously choose their profit-maximizing prices $\{p_{h1}^*, p_{l1}^*\}$. The equilibrium solution for package sizes and prices, derived through backwards induction, is summarized as follows:

In Stage 2 of the subgame, the firms set equilibrium package sizes $\{s_{h1}^*, s_{l1}^*\}$, such that

$$f'(s_{h1}^*) = \frac{k_h}{\theta q_h} \text{, and} \tag{27}$$

$$f'(s_{l1}^*) = \frac{k_l}{\theta q_l} . \tag{28}$$

In Stage 3 of the subgame, the firms set the following equilibrium prices:

$$p_{h1}^* = \frac{(4-\lambda)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}}{(8-5\lambda)} + k_h s_{h1}^* \text{, and} \tag{29}$$

$$p_{l1}^* = \frac{(3\lambda t - \lambda\alpha_{h1} + (4-3\lambda)\alpha_{l1})}{(8-5\lambda)} + k_l s_{l1}^* . \tag{30}$$

Given these equilibrium prices and package sizes, the corresponding equilibrium margins of the high- and low-quality products are $m_{h1}^* = \frac{((4-\lambda)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1})}{(8-5\lambda)}$ and $m_{l1}^* = \frac{(3\lambda t - \lambda\alpha_{h1} + (4-3\lambda)\alpha_{l1})}{(8-5\lambda)}$, respectively, and their corresponding equilibrium sales are $d_{h1}^* = \frac{\lambda((4-\lambda)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1})}{2t(8-5\lambda)}$ and $d_{l1}^* = \frac{(2-\lambda)(3\lambda t - \lambda\alpha_{h1} + (4-3\lambda)\alpha_{l1})}{2t(8-5\lambda)}$.

Therefore, the equilibrium profits of the products are

$$\Pi_{h1}^{*} = \frac{\lambda\left(\left(4-\lambda\right)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right)^{2}}{2t(8-5\lambda)^{2}}, \text{ and} \tag{31}$$

$$\Pi_{l1}^{*} = \frac{(2-\lambda)\left(3\lambda t - \lambda\alpha_{h1} + (4-3\lambda)\alpha_{l1}\right)^{2}}{2t(8-5\lambda)^{2}}, \tag{32}$$

where $\alpha_{h1} = \theta q_h f(s_{h1}^{*}) - k_h s_{h1}^{*}$, and $\alpha_{l1} = \theta q_l f(s_{l1}^{*}) - k_l s_{l1}^{*}$. In turn, the equilibrium *base values* of H and L are

$$\beta_{h1} = \theta q_h f(s_{h1}^{*}) - p_{h1}^{*} = \frac{2(2-\lambda)\alpha_{h1} + (2-\lambda)\alpha_{l1} - \left(4-\lambda\right)t}{(8-5\lambda)}, \text{ and} \tag{33}$$

$$\beta_{l1} = \theta q_l f(s_{l1}^{*}) - p_{l1}^{*} = \frac{\lambda\alpha_{h1} + 2(2-\lambda)\alpha_{l1} - 3\lambda t}{(8-5\lambda)}. \tag{34}$$

The *base value* of the high-quality product is relatively greater than that of the low-quality product $\left(\beta_{h1} > \beta_{l1}\right)$ if $\left(\alpha_{h1} > \alpha_{l1}\right)$.

When the high-quality firm does not sell to cash-constrained consumers, certain conditions are sufficient for the price of the high-quality product to exceed the price of the low-quality product. According to equations (29) and (30), the difference in equilibrium prices is

$$p_{h1}^{*} - p_{l1}^{*} = \frac{4\left(1-\lambda\right)t + 2\left(2-\lambda\right)\alpha_{h1} - 2(3-2\lambda)\alpha_{l1}}{(8-5\lambda)} + \left(k_h s_{h1}^{*} - k_l s_{l1}^{*}\right). \tag{35}$$

In the emerging market, the high-quality product is more expensive than the low-quality product $(p_{h1}^{*} > p_{l1}^{*})$, if $\left(\alpha_{h1} > \alpha_{l1}\right)$ and $\left(k_h s_{h1}^{*} > k_l s_{l1}^{*}\right)$, analogous to the developed market.

Furthermore, the unit transportation cost *t* is bound, as follows:

$$\frac{\alpha_{h1} - \alpha_{l1}}{3} < t < \frac{2\alpha_{h1} + \alpha_{l1}}{6}. \tag{36}$$

The lower bound arises because the margins must be positive, $\left(m_{h1}^{*} > 0\right)$ and $\left(m_{l1}^{*} > 0\right)$. The upper bound exists because consumers who purchase L must derive a positive utility from their purchase.

The following proposition compares the package sizes set by the firms in the emerging market, relative to the benchmark developed market.

Proposition 1: *When the high quality firm sells exclusively to the unconstrained segment in the emerging market, each firm sets the same package size in the emerging market, as compared to its package size in the benchmark market, i.e.* $s_{h1}^{*} = s_{h}^{*}$, *and* $s_{l1}^{*} = s_{l}^{*}$.

As Proposition 1 states, the firms' package sizing decision is unaffected by the cash constraint of consumers with a limited ability to pay. The high-quality firm is unaffected because it does not sell to cash-constrained consumers. In contrast, the low-quality firm is unaffected because its equilibrium price is less than the consumers' maximum ability to pay. Both firms set the same equilibrium package sizes as in the benchmark case, as indicated by the first-order conditions in the firms' profit-maximization problem. Next, the equilibrium prices set by the firms in the presence of cash-constrained consumers can be compared with the equilibrium prices in the benchmark model.

> **Proposition 2:** *When the high quality firm sells exclusively to the unconstrained segment in the emerging market, each firm sets a higher price in the emerging market, as compared to its price in the benchmark, developed market, i.e.* $p_{h1}^* > p_h^*$ *and* $p_{l1}^* > p_l^*$.

The intuition behind Proposition 2 is as follows: In the cash-constrained segment, no consumer can purchase the high-quality product because it is unaffordable. The high-quality firm compensates for this lack of sales by increasing the price it charges unconstrained consumers. At the same time, the low-quality firm enjoys a monopoly advantage in the cash-constrained segment and takes advantage of it by raising its price.

Propositions 1 and 2 collectively indicate that when the high-quality firm sells exclusively to an unconstrained market segment in the presence of cash-constrained consumers, both firms maintain the same package size but sell for relatively higher prices. Since the equilibrium package sizes remain unchanged, the *values* of the high- and low-quality products remain the same as in the benchmark case, or $(\alpha_{h1} = \alpha_h)$, and $(\alpha_{l1} = \alpha_l)$. The equilibrium prices are higher, so the base values of the high- and low-quality products are lower relative to the benchmark case, or $(\beta_{h1} < \beta_h)$, and $(\beta_{l1} < \beta_l)$.

Now compare the purchase behavior of consumers, based on their location on the Hotelling lines. In the benchmark case, consumers located between $(0 < x < x_0)$ in both segments purchase the high-quality product H, whereas those located between $(x_0 < x < 1)$ purchase the low-quality product L. In contrast, when the high-quality firm sells exclusively to the unconstrained segment, unconstrained consumers located between $(0 < x < x_{u1})$ purchase the high-quality product, whereas those located between $(x_{u1} < x < 1)$ purchase the low-quality product. In the cash-constrained segment, nobody purchases the (unaffordable) high-quality product, but some consumers purchase the low-quality product. Specifically, consumers located between $(0 < x < x_{c1})$ do not purchase anything, and consumers located between $(x_{c1} < x < 1)$ purchase the low-quality product.

The locations of the indifferent consumers on the Hotelling lines are related as $(x_{c1} < x_0 < x_{u1})$. This indicates the impact on the sales of the high- and low-quality products. The sales of the high-quality product in the unconstrained segment increase, because $(x_{u1} > x_0)$, but the sales of the low-quality product decrease, because $(1 - x_{u1} < 1 - x_0)$, relative to the benchmark case. Also, the sales of the low-quality product in the cash-constrained segment increase, because $(1 - x_{c1} > 1 - x_0)$, relative to the in benchmark case. The sales of the high-quality product in the cash-constrained segment relative to the benchmark case decrease by default, because the high-quality product is unaffordable for this segment.

It is interesting to note that the sales of the low-quality firm in the cash-constrained segment increase, despite the increase in its equilibrium price. Cash-constrained consumers are forced to choose between purchasing the low-quality product at the higher price or not making a purchase, so the low-quality firm can take advantage of the cash-constrained consumers' inability to afford the high-quality product.

Meanwhile, the relatively higher price of the low-quality product decreases its sales in the unconstrained segment, because some marginal consumers switch from purchasing the low-quality product to purchasing the high-quality product. This shift happens even though the prices of both products increase. In turn, sales of the high-quality product increase among the unconstrained segment.

The next proposition considers the impact of the cash-constrained segment on the profit made by the low-quality firm.

Proposition 3: *When the high quality firm sells exclusively to the unconstrained segment in the emerging market, the low-quality firm makes a higher profit in the emerging market, as compared to its profit in the benchmark, developed market, i.e.* $\Pi_{l1}^* > \Pi_l^*$.

It follows that the equilibrium margin of the low-quality firm is relatively higher in the emerging market, because its equilibrium price is higher and its package size, and hence its marginal cost, remain unchanged. Moreover, the low-quality firm enjoys a monopoly advantage in selling to the cash-constrained segment. Even though the sales of the low-quality product among unconstrained consumers decrease, total sales of the low-quality product across both market segments increase. Collectively, the low-quality firm makes a relatively larger profit, as summarized in Proposition 3.

This concludes the analysis when the high quality firm exclusively sells to the unconstrained market segment. In the next part of the analysis, the high-quality firm sells to both the cash-constrained and unconstrained market segments by setting a price that matches the limited ability to pay of the cash-constrained segment $\left(p_{h2}^{*} = \chi\right)$.

3.2.2 High-quality firm sells to both Unconstrained and Cash-Constrained Segments in the Emerging Market

In the emerging market, suppose the high-quality firm sets a price that matches the limited ability to pay of the cash-constrained market segment. Let $p_{h2} = \chi$ and s_{h2} represent the price and package size set by the high-quality firm when it sells to both the segments. Let $p_{l2} < \chi$ and s_{l2} represent the corresponding price and package size set by the low-quality firm. To summarize, $p_{l2} < \chi = p_{h2}$.

The consumers' utility functions are

$$U_{h2}(x) = \beta_{h2} - tx \text{, and} \tag{37}$$

$$U_{l2}(x) = \beta_{l2} - t(1-x) \text{,} \tag{38}$$

where β_{h2} and β_{l2} are the *base values* of the high- and low-quality products, defined respectively as

$$\beta_{h2} = \theta q_h f(s_{h2}) - \chi_c > 0 \text{, and} \tag{39}$$

$$\beta_{l2} = \theta q_l f(s_{l2}) - p_{l2} > 0 \text{.} \tag{40}$$

Consider the purchase behavior of the unconstrained segment of relative size λ. An unconstrained consumer located at x purchases H if $U_{h2}(x) > U_{l2}(x)$ and $U_{h2}(x) > 0$. Alternately, this consumer purchases L if $U_{l2}(x) > U_{h2}(x)$ and $U_{l2}(x) > 0$. Every unconstrained consumer in the market purchases a single unit of either H or L, and no unconstrained consumer purchases both H and L. Then let $x = x_2$ indicate the location of an unconstrained consumer indifferent between purchasing $\{H, L\}$, in which case $U_{h2}(x_{u2}) = U_{l2}(x_{u2})$, or $x_2 = \frac{(\beta_{h2} - \beta_{l2} + t)}{2t}$. Unconstrained consumers located between $(0 < x < x_2)$ purchase the high-quality product H; unconstrained consumers located between $(x_2 < x < 1)$ purchase the low-quality product L.

Now consider the cash-constrained segment of relative size $(1-\lambda)$. Since the price of H becomes affordable, $(p_{h2} = \chi)$, and cash-constrained consumers again choose between purchasing H or L. Cash-constrained consumers located between $(0 < x < x_2)$ purchase H, whereas cash-constrained consumers located between $(x_2 < x < 1)$ purchase L.

Let $\{d_{h2}, d_{l2}\}$ denote the aggregate sales of $\{H, L\}$ across the unconstrained and cash-constrained segments respectively. Then, $d_{h2} = \lambda x_2 + (1-\lambda) x_2 = x_2$ and $d_{l2} = \lambda(1-x_2) + (1-\lambda)(1-x_2) = (1-x_2)$. Since the marginal costs of $\{H, L\}$ are $c_{h2} = k_h s_{h2}$ and $c_{l1} = k_l s_{l2}$, the margins of $\{H, L\}$ are, respectively, $m_{h2} = (\chi_c - k_h s_{h2})$ and $m_{l2} = (p_{l2} - k_l s_{l2})$. In turn, the profit functions of $\{H, L\}$ are $\Pi_{h2} = m_{h2} d_{h2}$ and $\Pi_{l2} = m_{l2} d_{l2}$, yielding

$$\Pi_{h2} = (\chi_c - k_h s_{h2})\left(\frac{\beta_{h2} - \beta_{l2} + t}{2t}\right), \text{ and} \tag{41}$$

$$\Pi_{l2} = (p_{l2} - k_l s_{l2})\left(1 - \frac{\beta_{h2} - \beta_{l2} + t}{2t}\right), \tag{42}$$

where $\beta_{h2} = \theta q_h f(s_{h2}) - \chi_c > 0$, and $\beta_{l2} = \theta q_l f(s_{l2}) - p_{l2} > 0$.

Analogous to the benchmark model, the *values* of the high- and low-quality products equal the sum of their *base values* and margins, $\alpha_{h2} = \beta_{h2} + m_{h2}$ and $\alpha_{l2} = \beta_{l2} + m_{l2}$, which yields

$$\alpha_{h2} = \theta q_h f(s_{h2}^*) - k_h s_{h2}^* > 0 \text{, and} \tag{43}$$

$$\alpha_{l2} = \theta q_l f(s_{l2}^*) - k_l s_{l2}^* > 0 . \tag{44}$$

Furthermore, $(\alpha_{h2} > \beta_{h2} > 0)$ and $(\alpha_{l2} > \beta_{l2} > 0)$; otherwise, the firms will not sell their products.

Consider the Stage 2 and Stage 3 subgames. In Stage 2, both firms $\{H, L\}$ simultaneously choose their profit-maximizing package sizes $\{s_{h2}^*, s_{l2}^*\}$. In Stage 3, the high-quality firm sets its price as $p_{h2}^* = \chi$, and the low-quality firm sets its profit-maximizing price p_{l2}^*. Solving this subgame by backwards induction to determine the firms' equilibrium package sizes and prices provides the equilibrium solution summarized below.

In Stage 2, the high- and low-quality firms set package sizes that satisfy

$$f'(s_{h2}^*) = \frac{k_h}{\theta q_h} \frac{\left(3t + \beta_{h2} - \alpha_{l2}\right)}{\left(\chi - k_h s_{h2}^*\right)}, \text{ and} \tag{45}$$

$$f'(s_{l2}^*) = \frac{k_l}{\theta q_l}. \tag{46}$$

In Stage 3, the firms set the following equilibrium prices:

$$p_{h2}^* = \chi, \text{ and} \tag{47}$$

$$p_{l2}^* = \frac{t - \beta_{h2} + \alpha_{l2}}{2} + k_l s_{l2}^*, \tag{48}$$

where $\beta_{h2} = \theta q_h f(s_{h2}^*) - \chi$, and $\alpha_{l2} = \theta q_l f(s_{l2}^*) - k_l s_{l2}^*$.

With these equilibrium prices and package sizes, the corresponding equilibrium margins of H and L are $m_{h2}^* = \left(\chi - k_h s_{h2}^*\right)$ and $m_{l2}^* = \dfrac{\left(t - \beta_{h2} + \alpha_{l2}\right)}{2}$, respectively. The equilibrium sales are $d_{h2}^* = \dfrac{\left(3t + \beta_{h2} - \alpha_{l2}\right)}{4t}$ and $d_{l2}^* = \dfrac{\left(t - \beta_{h2} + \alpha_{l2}\right)}{4t}$, and the respective corresponding equilibrium profits are

$$\Pi_{h2}^* = \frac{\left(\chi - k_h s_{h2}^*\right)\left(3t + \beta_{h2} - \alpha_{l2}\right)}{4t}, \text{ and} \tag{49}$$

$$\Pi_{l2}^* = \frac{(t - \beta_{h2} + \alpha_{l2})^2}{8t}, \tag{50}$$

where $\alpha_{h2} = \theta q_h f(s_{h2}^*) - k_h s_{h2}^* > 0$, and $\alpha_{l2} = \theta q_l f(s_{l2}^*) - k_l s_{l2}^* > 0$.

When the high-quality firm sells to both segments in the emerging market, the conditions sufficient for the price of the high-quality product to exceed the price of the low-quality product arise from the following difference in equilibrium prices:

$$p_{h2}^{*} - p_{l2}^{*} = \frac{\left(\theta q_h f'(s_{h2}^{*}) + 2k_h\right)\left(\alpha_{h2} - \alpha_{l2}\right) + 2\left(\theta q_h f'(s_{h2}^{*}) + k_h\right)\left(k_h s_{h2}^{*} - k_l s_{l2}^{*}\right) + t\left(2k_h - \theta q_h f'(s_{h2}^{*})\right)}{2\left(k_h + \theta q_h f'(s_{h2}^{*})\right)} \quad (50)$$

In the emerging market, the high-quality product is more expensive than the low-quality product $(p_{h2}^{*} > p_{l2}^{*})$ if $\left(\alpha_{h2} > \alpha_{l2}\right)$ and $\left(k_h s_{h2}^{*} > k_l s_{l2}^{*}\right)$, analogous to the benchmark, developed market.

When the high-quality firm sells to both market segments, the unit transportation cost *t* is bound as

$$\beta_{h2} - \alpha_{l2} < t < \beta_{h2} + \frac{\alpha_{l2}}{3}. \quad (51)$$

The lower bound reflects that the margins must be positive, i.e. $m_{h2}^{*} > 0$ and $m_{l2}^{*} > 0$. The upper bound arises because consumers who purchase H and L must derive a positive utility from their purchases. To compare the package sizes set by the firms, relative to the benchmark model, the following proposition emerges.

Proposition 4: *When the high-quality firm sells to both segments in the emerging market, the high quality firm sets a smaller package size in the emerging market, as compared to its package size in the benchmark, developed market, i.e.* $s_{h2}^{*} < s_{h}^{*}$. *In contrast, the low quality firm sets the same package size in both markets, i.e.* $s_{l2}^{*} = s_{l}^{*}$.

Proposition 4 indicates that when the high-quality firm lowers its price to sell to both the market segments, it leads to a corresponding decrease in its equilibrium package size. In contrast, the low-quality firm remains unaffected by consumers' cash constraints, so its optimal package size also remains unchanged. This result follows from the first-order conditions of the firms' profit-maximization problem.

The smaller package size and price of the high-quality firm adversely affects the *base value* of the high quality product. When the high-quality firm sells to both segments in the emerging market, the *base value* of the high quality product in the emerging market, is smaller than its base value in the benchmark, developed market, i.e. $\beta_{h2} < \beta_{h}$. The limited ability to pay among cash-constrained consumers forces the high-quality firm to set a sub-optimal price than it would set in the absence of such customers. The lower price is accompanied by a corresponding smaller package size. This change causes the equilibrium *base value* offered by the high-quality product to decrease. The analysis next compares the equilibrium price set by the low-quality firm in the emerging market, with its price in the benchmark, developed market.

Proposition 5: *When the high-quality firm sells to both segments in the emerging market, the low-quality firm sets a higher price in the emerging market, as compared to its price in the benchmark, developed market, i.e.* $p_{l2}^{*} > p_{l}^{*}$.

It is optimal for the low-quality firm to set a higher price relative to its benchmark price. This is because the base value of the high quality product is lower in the emerging market. This reduces the utility that consumers get from purchasing the high-quality product. The low-quality firm takes advantage of this by raising its own price.

Although the high-quality firm decreases its price and package size in the emerging market, the low-quality firm maintains the same package size, but sets a relatively higher price. Since its package size is unchanged, the *value* of the low-quality product also remains the same as that in the benchmark developed market, i.e. $\alpha_{l2} = \alpha_{l}$. Furthermore, since the equilibrium price of the low-quality product is relatively higher, its *base value* decreases relative to the benchmark developed market, i.e. $\beta_{l2} < \beta_{l}$.

Now compare the purchase behavior of consumers based on their location on the Hotelling lines. The analysis reveals that the locations of indifferent consumers on the Hotelling lines are related as $(x_2 < x_0)$. This implies that the sales of the high-quality product decline as compared to the benchmark developed market, primarily because the *base value* of the high-quality product is lower, causing some consumers to switch from purchasing H to purchasing L.

The $(x_2 < x_0)$ relationship also enables a comparison of the sales of the low-quality product. The sales of L in the emerging and developed markets are $d_{l2} = (1 - x_2)$ and $d_{l} = (1 - x_0)$ respectively. Since $(1 - x_2) > (1 - x_0)$, the sales of L increase when the high-quality firm sells to both the cash-constrained and unconstrained segments compared with the benchmark case. As Proposition 5 reveals, the price of the low-quality product is relatively higher $\left(p_{l2}^{*} > p_{l}^{*}\right)$. However the marginal cost of the low-quality product remains the same, because Proposition 4 reveals that its package size remains unchanged $\left(s_{l2}^{*} = s_{l}^{*}\right)$. It then follows that the margin of L is relatively higher in the emerging market, as compared with to the benchmark developed case. In turn, the impact on its profit becomes clear.

Proposition 6: *When the high-quality firm sells to both segments in the emerging market, the low-quality firm earns a higher profit in the emerging market, as compared to its profit in the benchmark, developed market, i.e.* $\Pi_{l2}^{*} > \Pi_{l}^{*}$.

The analysis next determines when it is more profitable for the high-quality firm to serve one or two market segments.

3.2.3 When is it profitable for the high-quality firm to sell to both segments in the Emerging Market?

When the emerging market consists of one unconstrained segment and another cash-constrained segment, the high-quality firm decides to either serve only the unconstrained segment or alternately lower its price to match the limited ability to pay of the cash-constrained segment and sell to both segments. Recall that Π_{h1}^{*} and Π_{h2}^{*} represent the profits of the high-quality firm when it sells to one or two segments, respectively.

Proposition 7 establishes a relationship between the unit transportation cost and the high-quality firm's decision to sell to either one or both market segments. Recall that the unit transportation cost parameter $t > 0$ indicates the importance (unimportance) of preference (price), as well as a measure of consumers' price insensitivity (Kim et al. 2001).

Proposition 7: *The profit of the high quality firm from selling to both segments in the emerging market exceeds its profit from exclusively selling to the unconstrained segment in the emerging market i.e.* $\Pi_{h2}^{*} > \Pi_{h1}^{*}$, *when consumers' price sensitivity is relatively low, i.e.* $t > t_c$. *In contrast, if consumers' price sensitivity is relatively high, i.e.* $t \le t_c$, *the profit from exclusively selling to the unconstrained segment in the emerging market is relatively higher, i.e.* $\Pi_{h2}^{*} \le \Pi_{h1}^{*}$. *The threshold level of price sensitivity is*

$$t_c = \frac{m - \sqrt{m^2 - n}}{4\lambda(4-\lambda)^2}, \qquad (52)$$

where

$$m = 3(8-5\lambda)^2\left(\chi - k_h s_{h2}\right) - 4\lambda(4-\lambda)\left((4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right)$$

$$n = 8\lambda(4-\lambda)^2\left(2\lambda\left((4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right)^2 - (8-5\lambda)^2\left(\chi - k_h s_{h2}\right)\left(\beta_{h2} - \beta_l\right)\right)$$

3.3 Consumer Surplus

To measure and compare the consumer surplus in the emerging market and the benchmark, developed market, first consider the benchmark model with only unconstrained consumers. The surplus of all consumers who purchase the high-quality product is $CS_h = \int_0^{x_0} U_h(x)dx = \beta_h x_0 - \frac{t}{2}x_0^2$, and that of all consumers who purchase the low-quality product is $CS_l = \int_{x_0}^{1} U_l(x)dx = \left(\beta_l - t\right)(1-x_0) + \frac{t}{2}\left(1-x_0\right)^2$. At equilibrium, the surplus of consumers who purchase the high-quality product is lower than that of consumers who purchase the low-quality product $\left(CS_h < CS_l\right)$. Also, the aggregate consumer surplus is $CS = CS_h + CS_l$, or $CS = \left(2t - \beta_h - \beta_l\right)x_0 + \left(\beta_l - \frac{t}{2}\right)$, where $x_0 = \frac{3t + \alpha_h - \alpha_l}{6t}$.

Next, consider an emerging market with an unconstrained segment and a cash-constrained segment. The consumer surplus when the high-quality firm exclusively sells to the unconstrained segment is $\lambda\int_0^{x_{u1}} U_{h1}(x)dx = \lambda\left(\beta_{h1}x_{u1} - \frac{t}{2}x_{u1}^2\right)$ for those who purchase the high-quality product. Since no cash-constrained consumer can afford to purchase the high quality product, the aggregate surplus of all consumers who purchase the high-quality product is $CS_{h1} = \lambda\left(\beta_{h1}x_{u1} - \frac{t}{2}x_{u1}^2\right)$. Similarly, the surplus of consumers who purchase the low-quality product in the unconstrained segment is $\lambda\int_{x_{u1}}^{1} U_{l1}(x)dx = \lambda\left(\left(\beta_{l1} - t\right)(1-x_{u1}) + \frac{t}{2}\left(1-x_{u1}\right)^2\right)$, and the surplus of those who purchase the low-quality product in the cash-constrained segment is $\left(1-\lambda\right)\int_{x_{c1}}^{1} U_{l1}(x)dx = \left(1-\lambda\right)\left(\left(\beta_{l1} - t\right)(1-x_{c1}) + \frac{t}{2}\left(1-x_{c1}\right)^2\right)$. Thus, the aggregate surplus of all consumers who purchase the low-quality product is $CS_{l1} = \beta_{l1}\left(1 - \lambda x_{u1} - \left(1-\lambda\right)x_{c1}\right) - \frac{t}{2}\left(1 - \lambda x_{u1}^2 - \left(1-\lambda\right)x_{c1}^2\right)$. Finally, the aggregate consumer surplus of all consumers in the market, given by $CS_1 = CS_{h1} + CS_{l1}$, is

$CS_1 = \beta_{h1}\lambda x_{u1} + \beta_{l1}\left(1-\lambda x_{u1} - (1-\lambda)\, x_{c1}\right) - \frac{t}{2}\left(1-(1-\lambda)\, x_{c1}^2\right)$, where $x_{u1} = \frac{t+\beta_{h1}-\beta_{l1}}{2t}$, and $x_{c1} = \frac{t-\beta_{l1}}{t}$.

Comparing the consumer surplus when the high-quality firm H does not sell to the segment of cash-constrained consumers with the benchmark case in which no consumers are cash constrained reveals that each market segment may be divided into three sub-segments. In unconstrained segment, the change in the consumer surplus of each sub-segment is as follows:

1. Consumers located between $(0 < x < x_0)$ in the unconstrained segment purchase H in the benchmark case and continue to purchase H. However, they are worse off because the price of H is higher and its package size is the same; In other words, their surplus reduces since the *value* provided by H decreases.
2. Consumers located between $(x_0 < x < x_{u1})$ in the unconstrained segment switch from purchasing L in the benchmark case to purchasing H. Their consumer surplus decreases.
3. Consumers located between $(x_{u1} < x < 1)$ in the unconstrained segment purchase L in the benchmark case and continue to purchase L. However, they are worse off because the price of L is higher and its package size is the same; in other words, the value provided by L decreases.

This analysis therefore implies that when the high-quality firm H does not sell to a segment of cash-constrained consumers, the unconstrained segment collectively suffers. For the cash-constrained segment, the change in the consumer surplus of each sub-segment is as follows:

1. Consumers located between $(0 < x < x_{c1})$ in the cash-constrained segment purchase H in the benchmark case but do not purchase either product otherwise. They are worse off because they derived positive surplus from purchasing in the benchmark case and now attain no surplus.
2. Consumers located between $(x_{c1} < x < x_0)$ in the cash-constrained segment switch from purchasing H in the benchmark case to purchasing L. Their consumer surplus decreases.
3. Consumers located between $(x_0 < x < 1)$ in the cash-constrained segment purchase L in the benchmark case and continue to purchase L. They are worse off because the price of L is higher and its package size is the same; In other words, their surplus reduces since the *value*

provided by L decreases.

To summarize, when the high-quality firm H does not sell to the segment of cash-constrained consumers in an emerging market, the cash-constrained segment collectively suffers. The unconstrained segment in the emerging market also collectively suffers. Since both segments are worse off, the aggregate consumer surplus decreases, as stated in Proposition 8:

Proposition 8: *When the high quality firm sells exclusively to the unconstrained segment in the emerging market, the consumer surplus in the emerging market is lower than the consumer surplus in the benchmark, developed market, i.e.* $CS_1^* < CS_0^*$.

Next, the analysis considers the consumer surplus when the high-quality firm sells to both the segments in an emerging market. The surplus of consumers who purchase the high-quality product in the unconstrained segment is $\lambda\int_0^{x_2} U_{h2}(x)dx$, whereas that of consumers who purchase H in the cash-constrained segment is $\left(1-\lambda\right)\int_0^{x_2} U_{h2}(x)dx$. Thus, the aggregate consumer surplus of consumers who purchase the high-quality product is $CS_{h2} = \beta_{h2}x_2 - \frac{t}{2}x_2^2$. Similarly, the surplus of consumers who purchase the low-quality product in the unconstrained segment is $\lambda\int_{x_2^*}^{1} U_{l2}^*(x)dx$, whereas the surplus of consumers who purchase L in the cash-constrained segment is $\left(1-\lambda\right)\int_{x_2}^{1} U_{l2}(x)dx$. The aggregate consumer surplus of consumers who purchase the low-quality product is $CS_{l2} = \left(\beta_{l2}-t\right)(1-x_2) + \frac{t}{2}\left(1-x_2\right)^2$, where $x_2 = \frac{\beta_{h2}-\beta_{l2}+t}{2t}$. The aggregate surplus is $CS_2 = CS_{h2} + CS_{l2}$ is $CS_2 = \beta_{h2}x_2 - \frac{t}{2}x_2^2 + \left(\beta_{l2}-t\right)(1-x_2) + \frac{t}{2}\left(1-x_2\right)^2$.

When the high-quality firm sets a lower price to sell to the cash-constrained segment in the emerging market, consumer surplus differs from the benchmark case in which there are no cash-constrained consumers in the market. In the benchmark case, consumers located between $\left(0 < x < x_0\right)$ in both segments purchase H, whereas those located between $\left(x_0 < x < 1\right)$ purchase from the low-quality firm L. When one segment is cash constrained, consumers located between $\left(0 < x < x_2\right)$ in both segments purchase H, and those located between $\left(x_2 < x < 1\right)$

purchase L. Since $x_2 < x_0$, in comparing the purchase behavior of consumers, the market again divides into three sub-segments:

1. Consumers located between $(0 < x < x_2)$ in both segments continue to purchase H. They are worse off because the *value* provided by H is lower.
2. Consumers between $(x_2 < x < x_0)$ in both segments switch from purchasing from the high-quality firm H to purchasing from the low-quality firm L. Again, they are also worse off from this switch.
3. Consumers located between $(x_0 < x < 1)$ in both segments continue to purchase L. They are worse off because L raises its price $(p_{l2} > p_l)$ and keeps the same package size $(s_{l2} = s_l)$.

Overall, when the high-quality firm H lowers its price to serve cash-constrained consumers, consumer welfare declines, summarized as follows:

> **Proposition 9:** *When the high-quality firm sells to both segments in the emerging market, the consumer surplus in the emerging market is lower than the consumer surplus in the benchmark, developed market, i.e.* $CS_2^* < CS^*$.

4. Discussion and Conclusion

The purpose of this paper is to investigate package sizing decisions in an emerging market with cash-constrained consumers, relative to a developed market without cash-constrained consumers. Emerging markets are characterized by the presence of cash-constrained consumers with limited ability to pay, which restricts the prices that global manufacturers can charge in such markets. For example, most consumers in emerging markets cannot afford a $3 tube of Colgate toothpaste or a $5 bottle of Head & Shoulders shampoo, as commonly sold in a developed market such as the United States. One approach adopted by global manufacturers to overcome this limitation is to sell their products in smaller package sizes in emerging markets.

The 2007 annual category sales of shampoo and bathing soap in India highlight this issue: According to A.C. Nielsen, India, in the shampoo category, 50 ml and 100 ml bottles of shampoo were sold for average prices of $1 and $1.75, respectively, and collectively accounted for 73% of the 2007 revenue from shampoo sales in the country. In contrast, the typical bottle of shampoo sold by Wal-Mart in the United States is 350 ml or larger. Similarly, research by A.C. Nielsen, India, indicates that 65% of the 2007 category revenue from bathing soap sales in India came from bars of soap weighing 70–90 grams and selling for $0.22 on average; the typical bar of soap sold in U.S. Wal-Marts weighs 150 grams and is sold in multi-packs.

This article analyzes the competitive and strategic implications of smaller package sizes in an emerging market. Summarizing the model, an emerging market has two horizontally and vertically differentiated firms, each selling a product to two market segments. A cash-constrained market segment cannot afford to purchase the high-quality, higher priced product, whereas a second unconstrained market segment can afford both products. For comparison, a benchmark, developed market has two unconstrained market segments. In the emerging market, the high-quality firm decides between serving the unconstrained market segment exclusively or lowering its price and selling to both market segments. Both firms then set their optimal package sizes and prices. The analysis compares the equilibrium package sizes and prices in the emerging and developed markets respectively.

The high-quality firm sells its product to both market segments by lowering its price and the corresponding package size, provided consumers' price sensitivity is not too high. In this case, the low-quality firm maintains the same package size, but raises its price, relative to the benchmark developed market.

The high-quality firm alternately exclusively sells to the unconstrained segment if consumers' price sensitivity is sufficiently high. Cash-constrained consumers cannot afford the high-quality product and choose between purchasing the low-quality product or not making a purchase. In this case, both firms leave their package sizes relatively unchanged and charge a relatively higher price in the emerging market compared with the benchmark developed market.

Overall, the low quality product sells for a higher price in an emerging market with cash-constrained consumers, as compared to a benchmark market without cash-constrained consumers. This is true both when the high quality firm sells exclusively to the unconstrained segment, as well as when the high quality firms sells to both the unconstrained and cash-constrained segments. Also overall, the value that consumers obtain from the two products in the emerging market is comparatively lower, leading to a decline in consumer surplus.

In summary, this research has important implications for global manufacturers selling in both developed and emerging markets. This article normatively analyzes the package sizing and pricing decisions made by firms in emerging markets and highlights the interrelated strategic and competitive implications.

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Appendices

Appendix 1: Benchmark Model of a Developed Market with only Unconstrained Consumers

The firms' decision making takes place in two stages when both segments are unconstrained. In Stage 1, both firms $\{H, L\}$ simultaneously choose their profit maximizing package sizes $\{s_h^*, s_l^*\}$. In Stage 2, both firms $\{H, L\}$ simultaneously choose their profit maximizing prices $\{p_h^*, p_l^*\}$. We solve the sub-game by backwards induction, to determine the firms' equilibrium package sizes and prices.

The firms' profit functions are $\Pi_h = (p_h - k_h s_h)\left(\frac{\beta_h - \beta_l + t}{2t}\right)$,

$\Pi_l = (p_l - k_l s_l)\left(1 - \frac{\beta_h - \beta_l + t}{2t}\right)$, where $\beta_h = \theta q_h f(s_h) - p_h > 0$, $\beta_l = \theta q_l f(s_l) - p_l > 0$.

Consider the Stage 2 pricing sub-game:

In Stage 2, the firms set their equilibrium prices such that their marginal benefit equals their marginal cost. The first-order and second-order conditions are $\frac{\partial \Pi_h}{\partial p_h} = 0$, $\frac{\partial \Pi_l}{\partial p_l} = 0$, $\frac{\partial^2 \Pi_h}{\partial p_h^2} < 0$, $\frac{\partial^2 \Pi_l}{\partial p_l^2} < 0$. Differentiating the profit functions,

$$\frac{\partial \Pi_h}{\partial p_h} = \frac{-2p_h + p_l + k_h s_h + t + \theta\left(q_h f(s_h) - q_l f(s_l)\right)}{2t}$$

$$\frac{\partial \Pi_l}{\partial p_l} = \frac{p_h - 2p_l + k_l s_l + t - \theta\left(q_h f(s_h) - q_l f(s_l)\right)}{2t}$$

$$\frac{\partial^2 \Pi_h}{\partial p_h^2} = \frac{\partial^2 \Pi_l}{\partial p_l^2} = \frac{-1}{t} < 0$$

Solving the first-order conditions simultaneously, yields the following prices:

$$p_h = \frac{3t + \theta q_h f(s_h) - \theta q_l f(s_l) + 2k_h s_h + k_l s_l}{3}$$

$$p_l = \frac{3t - \theta q_h f(s_h) + \theta q_l f(s_l) + k_h s_h + 2k_l s_l}{3}$$

Now let us consider the Stage 1 package-sizing sub-game:

We substitute the equilibrium prices from Stage 2 back into the profit functions yielding

$$\Pi_h = \frac{\left(3t + \theta q_h f(s_h) - k_h s_h - \theta q_l f(s_l) + k_l s_l\right)^2}{18t}$$

$$\Pi_l = \frac{\left(3t - \theta q_h f(s_h) + k_h s_h + \theta q_l f(s_l) - k_l s_l\right)^2}{18t}$$

In Stage 1, the firms set their equilibrium package sizes such that their marginal benefit equals their marginal cost. The first-order and second-order conditions are $\frac{\partial \Pi_h}{\partial s_h} = 0$, $\frac{\partial \Pi_l}{\partial s_l} = 0$,

$\frac{\partial^2 \Pi_h}{\partial s_h^2} < 0$, $\frac{\partial^2 \Pi_l}{\partial s_l^2} < 0$. Differentiating the profit functions with respect to the package sizes respectively,

$$\frac{\partial \Pi_h}{\partial s_h} = \frac{\left(3t + \theta q_h f(s_h) - k_h s_h - \theta q_l f(s_l) + k_l s_l\right)\left(\theta q_h f'(s_h) - k_h\right)}{9t}$$

$$\frac{\partial \Pi_l}{\partial s_l} = \frac{\left(3t - \theta q_h f(s_h) + k_h s_h + \theta q_l f(s_l) - k_l s_l\right)\left(\theta q_l f'(s_l) - k_l\right)}{9t}$$

We get the equilibrium package sizes $\{s_h^*, s_l^*\}$ by simultaneously solving the first-order conditions $\frac{\partial \Pi_h}{\partial s_h} = 0$, $\frac{\partial \Pi_l}{\partial s_l} = 0$. Observe that $\left(3t + \theta q_h f(s_h) - k_h s_h - \theta q_l f(s_l) + k_l s_l\right) \neq 0$ and $\left(3t - \theta q_h f(s_h) + k_h s_h + \theta q_l f(s_l) - k_l s_l\right) \neq 0$. Otherwise, we would get $\Pi_h = 0$ and $\Pi_l = 0$ respectively. Thus, in Stage 1, the firms set equilibrium package sizes $\{s_h^*, s_l^*\}$, such that $\left(\theta q_h f'(s_h) - k_h\right) = 0$ and $\left(\theta q_l f'(s_l) - k_l\right) = 0$, yielding $f'(s_h^*) = \frac{k_h}{\theta q_h}$, $f'(s_l^*) = \frac{k_l}{\theta q_l}$.

In Stage 2, the firms set equilibrium prices $\{p_h^*, p_l^*\}$ as follows:

$$p_h^* = \frac{3t + \theta q_h f(s_h^*) - \theta q_l f(s_l^*) + 2k_h s_h^* + k_l s_l^*}{3}$$

$$p_l^* = \frac{3t - \theta q_h f(s_h^*) + \theta q_l f(s_l^*) + k_h s_h^* + 2k_l s_l^*}{3}$$

Substituting $\alpha_h = \theta q_h f(s_h^*) - k_h s_h^* > 0$, $\alpha_l = \theta q_l f(s_l^*) - k_l s_l^* > 0$, the equilibrium prices are

$$p_h^* = \frac{3t + \alpha_h - \alpha_l}{3} + k_h s_h^*, \quad p_l^* = \frac{3t - \alpha_h + \alpha_l}{3} + k_l s_l^*$$

The high quality product is more expensive than the low quality product $(p_h^* > p_l^*)$ if $\left(\alpha_h > \alpha_l\right)$ and $\left(k_h s_h^* > k_l s_l^*\right)$. The difference in prices is $p_h^* - p_l^* = \frac{2\left(\alpha_h - \alpha_l\right) + 3\left(k_h s_h^* - k_l s_l^*\right)}{3}$. It is clear that $\left(\alpha_h > \alpha_l\right)$ and $\left(k_h s_h^* > k_l s_l^*\right)$ are sufficient conditions to ensure that $p_h^* > p_l^*$.

Lemma 1: *In the benchmark, developed market, the high-quality firm sets a smaller package size than the low-quality firm, i.e.* $s_h^* < s_l^*$, *if* $k_h > \frac{q_h}{q_l} k_l$

Proof of Lemma 1: Since $f'(s_h^*) = \frac{k_h}{\theta q_h}$, $f'(s_l^*) = \frac{k_l}{\theta q_l}$, and since f is concave, $(s_h^* < s_l^*) \equiv f'(s_h^*) > f'(s_l^*) \equiv \frac{k_h}{k_l} > \frac{q_h}{q_l}$

QED

The corresponding equilibrium margins of H and L are $m_h^* = t + \frac{\alpha_h - \alpha_l}{3}$, $m_l^* = t - \frac{\alpha_h - \alpha_l}{3}$

The equilibrium sales of H and L are $d_h^* = \frac{1}{2} + \frac{\alpha_h - \alpha_l}{6t}$, $d_l^* = \frac{1}{2} - \frac{\alpha_h - \alpha_l}{6t}$.

The equilibrium profits of H and L are $\Pi_h^* = \frac{1}{2t}\left(t + \frac{\alpha_h - \alpha_l}{3}\right)^2$, $\Pi_l^* = \frac{1}{2t}\left(t - \frac{\alpha_h - \alpha_l}{3}\right)^2$, where

$$\alpha_h = \theta q_h f(s_h^*) - k_h s_h^* > 0, \; \alpha_l = \theta q_l f(s_l^*) - k_l s_l^* > 0.$$

Next, we consider the necessary bounds on the unit transportation cost t is bounded. Since the margins should be positive, $(m_h^* > 0)$, $(m_l^* > 0)$, we have $t > \frac{\alpha_h - \alpha_l}{3}$. Also, the utility that consumers get from purchasing the high or low quality product should be strictly positive, or $(x_{l0} < x_0 < x_{h0})$, or $(x_0 - x_{l0}) > 0$, $(x_{h0}^* - x_0^*) > 0$, where $x_{h0} = \frac{2\alpha_h + \alpha_l - 3t}{3t}$, $x_0 = \frac{3t + \alpha_h - \alpha_l}{6t}$, $x_{l0} = \frac{6t - \alpha_h - 2\alpha_l}{3t}$. Thus, $(x_0 - x_{l0}) = (x_{h0} - x_0) = \frac{\alpha_h + \alpha_l - 3t}{2t} > 0$

These inequalities yield $t < \frac{\alpha_h + \alpha_l}{3}$. Collectively, the unit transportation cost is bounded as

$\frac{\alpha_h - \alpha_l}{3} < t < \frac{\alpha_h + \alpha_l}{3}$.

The base value of the high quality product is relatively more than the low quality product, with $(0 < \beta_l < \beta_h < t)$. To see this, recall that the base values of H and L are $\beta_h = \theta q_h f(s_h^*) - p_h^*$ and $\beta_l = \theta q_l f(s_l^*) - p_l^*$. Since $\alpha_h = \theta q_h f(s_h^*) - k_h s_h^*$, we have $\theta q_h f(s_h^*) = \alpha_h + k_h s_h^*$. Since $p_h^* = \frac{3t + \alpha_h - \alpha_l}{3} + k_h s_h^*$, we get

$$\beta_h = \theta q_h f(s_h^*) - p_h^* = \alpha_h + k_h s_h^* - \frac{3t + \alpha_h - \alpha_l}{3} - k_h s_h^* = \frac{2\alpha_h + \alpha_l}{3} - t$$

Thus, we have $\beta_h = \theta q_h f(s_h^*) - p_h^* = \frac{2\alpha_h + \alpha_l}{3} - t$. And analogously, we also have $\beta_l = \theta q_l f(s_l^*) - p_l^* = \frac{\alpha_h + 2\alpha_l}{3} - t$. Notice that $\beta_h - \beta_l = \frac{\alpha_h - \alpha_l}{3}$.

Recall that the high quality product is relatively more expensive than the low quality product $(p_h^* > p_l^*)$ if $(\alpha_h > \alpha_l)$ and $(k_h s_h^* > k_l s_l^*)$. Thus, if H is more expensive than L, the base value of H is relatively higher than the base value of L, or $(\beta_h > \beta_l)$.

Also, since $x_{h0} = \frac{2\alpha_h + \alpha_l - 3t}{3t}$ and $\beta_h = \theta q_h f(s_h^*) - p_h^* = \frac{2\alpha_h + \alpha_l}{3} - t$, we get $3t(1 + x_{h0}) = 2\alpha_h + \alpha_l$ and thus $\beta_h = t x_{h0}$, or $x_{h0} = \frac{\beta_h}{t}$. Since $0 < x_{h0} < 1$, we get $\beta_h < t$. Collectively, it follows that $(0 < \beta_l < \beta_h < t)$.

The surplus of consumers who purchase H in the first unconstrained segment is $\lambda \int_0^{x_0} U_h(x)dx$, while the surplus of consumers who purchase H in the second unconstrained segment is

$\left(1-\lambda\right)\int_0^{x_0^*} U_h(x)dx$. Thus, the aggregate consumer surplus of consumers who purchase H is $CS_h = \int_0^{x_0} U_h(x)dx = \int_0^{x_0} \left(\beta_h - tx\right)dx$. This yields $CS_h = \beta_h x_0 - \frac{t}{2}\left(x_0\right)^2$. Similarly, the surplus of consumers who purchase L in the first unconstrained segment is $\lambda\int_{x_0}^1 U_l(x)dx$, while the surplus of consumers who purchase L in the second unconstrained segment is $\left(1-\lambda\right)\int_{x_0}^1 U_l(x)dx$. The aggregate consumer surplus of consumers who purchase L is $CS_l = \int_{x_0}^1 U_l(x)dx = \int_{x_0}^1 \left(\beta_l - t(1-x)\right)dx$. This yields $CS_l = \left(\beta_l - t\right)(1-x_0) + \frac{t}{2}\left(1-x_0\right)^2$ where $x_0 = \frac{3t+\alpha_h-\alpha_l}{6t}$. The aggregate consumer surplus $CS = CS_h + CS_l$ is $CS = \left(\beta_l - t\right)(1-x_0) + \frac{t}{2}\left(1-x_0\right)^2 + \beta_h x_0 - \frac{t}{2}\left(x_0\right)^2$, or $CS = \left(2t-\beta_h-\beta_l\right)x_0 + \left(\beta_l - \frac{t}{2}\right)$.

Appendix 2

Model of an Emerging Market with Unconstrained and Cash-Constrained Consumers: High-quality firm sells exclusively to the Unconstrained Segment:

The firms' profit functions are

$$\Pi_{h1} = \lambda\left(p_{h1} - k_h s_{h1}\right)\left(\frac{\beta_{h1}-\beta_{l1}+t}{2t}\right)$$

$$\Pi_{l1} = \left(p_{l1} - k_l s_{l1}\right)\left(1-\lambda\left(\frac{\beta_{h1}-\beta_{l1}+t}{2t}\right)-(1-\lambda)\left(\frac{t-\beta_{l1}}{t}\right)\right)$$

where $\beta_{h1} = \theta q_h f(s_{h1}) - p_{h1} > 0$, $\beta_{l1} = \theta q_l f(s_{l1}) - p_{l1} > 0$. We have

$$\Pi_{h1} = \left(p_{h1} - k_h s_{h1}\right)\lambda\left(\frac{\theta\left(q_h f(s_{h1}) - q_l f(s_{l1})\right) - \left(p_{h1} - p_{l1}\right) + t}{2t}\right)$$

$$\Pi_{l1} = \left(p_{l1} - k_l s_{l1}\right)\left(1-\lambda\left(\frac{\theta\left(q_h f(s_{h1}) - q_l f(s_{l1})\right) - \left(p_{h1} - p_{l1}\right) + t}{2t}\right)-(1-\lambda)\left(\frac{p_{l1} - \theta q_l f(s_{l1}) + t}{t}\right)\right)$$

In Stage 2 of this sub-game, both firms $\{H, L\}$ simultaneously choose their profit maximizing package sizes $\{s_{h1}^*, s_{l1}^*\}$. In Stage 3 of this sub-game, both the firms $\{H, L\}$ simultaneously choose their profit maximizing prices $\{p_{h1}^*, p_{l1}^*\}$. We solve the game by backwards induction, to determine the firms' equilibrium package sizes and prices.

Consider the Stage 3 pricing sub-game:

In Stage 3, the firms set their equilibrium prices such that their marginal benefit equals their marginal cost. The first-order and second-order conditions are $\frac{\partial \Pi_{h1}}{\partial p_{h1}} = 0$, $\frac{\partial \Pi_{l1}}{\partial p_{l1}} = 0$, $\frac{\partial^2 \Pi_{h1}}{\partial p_{h1}^2} < 0$,

$\frac{\partial^2 \Pi_{l1}}{\partial p_{l1}^2} < 0$. Solving the first-order conditions, we find that in Stage 3, the firms set equilibrium prices $\{p_{h1}^*, p_{l1}^*\}$ as follows:

$$p_{h1}^* = \frac{\left(2\left(2-\lambda\right)k_h s_{h1}^* + \left(2-\lambda\right)k_l s_{l1}^* + \left(4-\lambda\right)t + (4-3\lambda)\theta q_h f(s_{h1}^*) - (2-\lambda)\theta q_l f(s_{l1})\right)}{(8-5\lambda)}$$

$$p_{l1}^* = \frac{\left(2\left(2-\lambda\right)k_l s_{l1}^* + \lambda k_h s_{h1}^* + 3\lambda t - \lambda\theta q_h f(s_{h1}^*) + (4-3\lambda)\theta q_l f(s_{l1})\right)}{(8-5\lambda)}$$

For brevity, let us define α_{h1} and α_{l1} as $\alpha_{h1} = \theta q_h f(s_{h1}^*) - k_h s_{h1}^* > 0$, $\alpha_{l1} = \theta q_l f(s_{l1}^*) - k_l s_{l1}^* > 0$. We can rewrite the equilibrium prices as

$$p_{h1}^* = \frac{(\left(4-\lambda\right)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1})}{(8-5\lambda)} + k_h s_{h1}^*$$

$$p_{l1}^* = \frac{\left(3\lambda t - \lambda\alpha_{h1} + (4-3\lambda)\alpha_{l1}\right)}{(8-5\lambda)} + k_l s_{l1}^*$$

The corresponding equilibrium margins of H and L are

$$m_{h1}^* = \frac{(\left(4-\lambda\right)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1})}{(8-5\lambda)},\ m_{l1}^* = \frac{\left(3\lambda t - \lambda\alpha_{h1} + (4-3\lambda)\alpha_{l1}\right)}{(8-5\lambda)}$$

The corresponding equilibrium sales of H and L are

$$d_{h1}^* = \frac{\lambda(\left(4-\lambda\right)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1})}{2t(8-5\lambda)},\ d_{l1}^* = \frac{(2-\lambda)\left(3\lambda t - \lambda\alpha_{h1} + (4-3\lambda)\alpha_{l1}\right)}{2t(8-5\lambda)}$$

The corresponding equilibrium profits of H and L are

$$\Pi_{h1}^* = \frac{\lambda(\left(4-\lambda\right)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1})^2}{2t(8-5\lambda)^2},\ \Pi_{l1}^* = \frac{(2-\lambda)\left(3\lambda t - \lambda\alpha_{h1} + (4-3\lambda)\alpha_{l1}\right)^2}{2t(8-5\lambda)^2}$$

where $\alpha_{h1} = \theta q_h f(s_{h1}^*) - k_h s_{h1}^*$, $\alpha_{l1} = \theta q_l f(s_{l1}^*) - k_l s_{l1}^*$.

Proceeding with backward induction, consider the Stage 2 package-sizing sub-game. In Stage 2, the firms set their equilibrium package sizes such that their marginal benefit equals their marginal cost. The first-order and second-order conditions are $\frac{\partial \Pi_{h1}}{\partial s_{h1}} = 0$, $\frac{\partial \Pi_{l1}}{\partial s_{l1}} = 0$, $\frac{\partial^2 \Pi_{h1}}{\partial s_{h1}^2} < 0$, $\frac{\partial^2 \Pi_{l1}}{\partial s_{l1}^2} < 0$. Solving the first-order conditions, we find that in Stage 2, the firms set equilibrium package sizes $\{s_{h1}^*, s_{l1}^*\}$, such that $f'(s_{h1}^*) = \frac{k_h}{\theta q_h}$, $f'(s_{l1}^*) = \frac{k_l}{\theta q_l}$.

We will now prove that the unit transportation cost is bound as $\frac{\alpha_{h1} - \alpha_{l1}}{3} < t < \frac{2\alpha_{h1} + \alpha_{l1}}{6}$

Since the margins need to be positive, $\left(m_{h1}^* > 0\right)$, $\left(m_{l1}^* > 0\right)$, we have

$$t > \frac{(2-\lambda)\alpha_{l1} - (4-3\lambda)\alpha_{h1}}{\left(4-\lambda\right)},\ t > \frac{\lambda\alpha_{h1} - (4-3\lambda)\alpha_{l1}}{3\lambda}.$$

Since consumers who purchase L must derive a positive utility from their purchase, we require $\left(x_{c1}^* < x_{u1}^*\right)$. We have $x_{u1} - x_{c1} = \dfrac{((4-\lambda)\alpha_{h1} + 3(2-\lambda)\alpha_{l1} - 3\left(4-\lambda\right)t)}{2t(8-5\lambda)}$. $\left(x_{u1} - x_{c1}\right) > 0$ implies that t needs to be upper-bound as $t < \dfrac{(4-\lambda)\alpha_{h1} + 3(2-\lambda)\alpha_{l1}}{3\left(4-\lambda\right)}$. Collectively, we find that t is bound as

$$\frac{\lambda\alpha_{h1} - (4-3\lambda)\alpha_{l1}}{3\lambda} < t < \frac{(4-\lambda)\alpha_{h1} + 3(2-\lambda)\alpha_{l1}}{3\left(4-\lambda\right)}$$

Since $\left(0 < \lambda < 1\right)$, $Max\left(\dfrac{\lambda\alpha_{h1} - (4-3\lambda)\alpha_{l1}}{3\lambda}\right) = \dfrac{\alpha_{h1} - \alpha_{l1}}{3}$,

$$Min\left(\frac{(4-\lambda)\alpha_{h1} + 3(2-\lambda)\alpha_{l1}}{3\left(4-\lambda\right)}\right) = \frac{2\alpha_{h1} + \alpha_{l1}}{6}$$

This collectively implies that t is bound as $\dfrac{\alpha_{h1} - \alpha_{l1}}{3} < t < \dfrac{2\alpha_{h1} + \alpha_{l1}}{6}$.

Next, we establish the necessary conditions for the price of the high quality product to exceed the price of the low quality product. The high quality product is more expensive than the low quality product $(p_{h1}^* > p_{l1}^*)$, if $\left(\alpha_{h1} > \alpha_{l1}\right)$ and $\left(k_h s_{h1}^* > k_l s_{l1}^*\right)$. We find this is analogous to the benchmark model. The difference in equilibrium prices is

$$p_{h1}^* - p_{l1}^* = \frac{4\left(1-\lambda\right)t + 2\left(2-\lambda\right)\alpha_{h1} - 2(3-2\lambda)\alpha_{l1}}{(8-5\lambda)} + \left(k_h s_{h1}^* - k_l s_{l1}^*\right)$$

The high quality product is more expensive than the low quality product if $k_h s_{h1}^* > k_l s_{l1}^*$ and $4\left(1-\lambda\right)t + 2\left(2-\lambda\right)\alpha_{h1} - 2(3-2\lambda)\alpha_{l1} > 0$, or $t > \dfrac{2(3-2\lambda)\alpha_{l1} - 2\left(2-\lambda\right)\alpha_{h1}}{4\left(1-\lambda\right)}$.

Recall from our analysis above that t is constrained as

$$\frac{\lambda\alpha_{h1} - (4-3\lambda)\alpha_{l1}}{3\lambda} < t < \frac{(4-\lambda)\alpha_{h1} + 3(2-\lambda)\alpha_{l1}}{3\left(4-\lambda\right)}$$

We compare $t > \dfrac{2(3-2\lambda)\alpha_{l1} - 2\left(2-\lambda\right)\alpha_{h1}}{4\left(1-\lambda\right)}$ with the lower bound from

$$\frac{\lambda\alpha_{h1} - (4-3\lambda)\alpha_{l1}}{3\lambda} < t < \frac{(4-\lambda)\alpha_{h1} + 3(2-\lambda)\alpha_{l1}}{3\left(4-\lambda\right)}$$

Notice that $\left(\dfrac{\lambda\alpha_{h1} - (4-3\lambda)\alpha_{l1}}{3\lambda}\right) - \left(\dfrac{2(3-2\lambda)\alpha_{l1} - 2\left(2-\lambda\right)\alpha_{h1}}{4\left(1-\lambda\right)}\right)$

$$= \frac{\left(4\lambda\left(1-\lambda\right) + 6\lambda\left(2-\lambda\right)\right)\alpha_{h1} - \left(4\left(1-\lambda\right)(4-3\lambda) + 6\lambda(3-2\lambda)\right)\alpha_{l1}}{12\lambda\left(1-\lambda\right)} = \frac{2\left(8-5\lambda\right)\left(\lambda\alpha_{h1} - \alpha_{l1}\right)}{12\lambda\left(1-\lambda\right)}$$

It follows that $\left(\frac{\lambda\alpha_{h1}-(4-3\lambda)\alpha_{l1}}{3\lambda}\right)>\left(\frac{2(3-2\lambda)\alpha_{l1}-2(2-\lambda)\alpha_{h1}}{4(1-\lambda)}\right)$ if $(\lambda\alpha_{h1}>\alpha_{l1})$. And since $\alpha_{h1}>\lambda\alpha_{h1}$, we get $(\alpha_{h1}>\alpha_{l1})$. Collectively, we find that the high quality product is more expensive than the low quality product $(p^*_{h1}>p^*_{l1})$, if $(\alpha_{h1}>\alpha_{l1})$ and $(k_h s^*_{h1}>k_l s^*_{l1})$.

Let us now consider the equilibrium base values of H and L: $\beta_{h1}=\theta q_h f(s^*_{h1})-p^*_{h1}$, $\beta_{l1}=\theta q_l f(s^*_{l1})-p^*_{l1}$. Since $\alpha_{h1}=\theta q_h f(s^*_{h1})-k_h s^*_{h1}$, we have $\theta q_h f(s^*_{h1})=\alpha_{h1}+k_h s^*_{h1}$

And since $p^*_{h1}=\frac{((4-\lambda)t+(4-3\lambda)\alpha_{h1}-(2-\lambda)\alpha_{l1})}{(8-5\lambda)}+k_h s^*_{h1}$, we get

$$\begin{aligned}\beta_{h1}&=\theta q_h f(s^*_{h1})-p^*_{h1}\\&=\alpha_{h1}+k_h s^*_{h1}-\frac{((4-\lambda)t+(4-3\lambda)\alpha_{h1}-(2-\lambda)\alpha_{l1})}{(8-5\lambda)}-k_h s^*_{h1}\\&=\frac{(8-5\lambda)\alpha_{h1}-(4-\lambda)t-(4-3\lambda)\alpha_{h1}+(2-\lambda)\alpha_{l1}}{(8-5\lambda)}\\&=\frac{2(2-\lambda)\alpha_{h1}-(4-\lambda)t+(2-\lambda)\alpha_{l1}}{(8-5\lambda)}\end{aligned}$$

This yields $\beta_{h1}=\frac{2(2-\lambda)\alpha_{h1}+(2-\lambda)\alpha_{l1}-(4-\lambda)t}{(8-5\lambda)}$

Analogously, we have $p^*_{l1}=\frac{(3\lambda t-\lambda\alpha_{h1}+(4-3\lambda)\alpha_{l1})}{(8-5\lambda)}+k_l s^*_{l1}$ and we get

$$\begin{aligned}\beta_{l1}&=\theta q_l f(s^*_{l1})-p^*_{l1}\\&=\alpha_{l1}-\frac{(3\lambda t-\lambda\alpha_{h1}+(4-3\lambda)\alpha_{l1})}{(8-5\lambda)}\\&=\frac{(8-5\lambda)\alpha_{l1}-3\lambda t+\lambda\alpha_{h1}-(4-3\lambda)\alpha_{l1}}{(8-5\lambda)}\\&=\frac{2(2-\lambda)\alpha_{l1}-3\lambda t+\lambda\alpha_{h1}}{(8-5\lambda)}\end{aligned}$$

To summarize, the equilibrium base values of H and L are given as follows:

$$\beta_{h1}=\frac{2(2-\lambda)\alpha_{h1}+(2-\lambda)\alpha_{l1}-(4-\lambda)t}{(8-5\lambda)}$$

$$\beta_{l1}=\frac{\lambda\alpha_{h1}+2(2-\lambda)\alpha_{l1}-3\lambda t}{(8-5\lambda)}$$

We can now compare the equilibrium base values of H and L. We show that the base value of the high quality product is relatively more than the low quality product $(\beta_{h1}>\beta_{l1})$. We find this is analogous to the benchmark model.

Notice that

$$\beta_{h1}-\beta_{l1}=\frac{2(2-\lambda)\alpha_{h1}+(2-\lambda)\alpha_{l1}-\left(4-\lambda\right)t}{(8-5\lambda)}-\frac{\lambda\alpha_{h1}+2(2-\lambda)\alpha_{l1}-3\lambda t}{(8-5\lambda)}$$

$$\beta_{h1}-\beta_{l1}=\frac{(4-3\lambda)\alpha_{h1}-(2-\lambda)\alpha_{l1}-\left(1-\lambda\right)t}{(8-5\lambda)}$$

Since $\frac{\lambda\alpha_{h1}-(4-3\lambda)\alpha_{l1}}{3\lambda}<t<\frac{(4-\lambda)\alpha_{h1}+3(2-\lambda)\alpha_{l1}}{3\left(4-\lambda\right)}$,

$$\beta_{h1}-\beta_{l1}>\frac{(4-3\lambda)\alpha_{h1}-(2-\lambda)\alpha_{l1}-\left(1-\lambda\right)\frac{(4-\lambda)\alpha_{h1}+3(2-\lambda)\alpha_{l1}}{3(4-\lambda)}}{(8-5\lambda)}$$

$$=\frac{3\left(4-\lambda\right)(4-3\lambda)\alpha_{h1}-3\left(4-\lambda\right)(2-\lambda)\alpha_{l1}-\left(1-\lambda\right)\left((4-\lambda)\alpha_{h1}+3(2-\lambda)\alpha_{l1}\right)}{3\left(4-\lambda\right)(8-5\lambda)}$$

$$=\frac{\left(4-\lambda\right)\left(11-8\lambda\right)\alpha_{h1}-9(2-\lambda)\alpha_{l1}}{3\left(4-\lambda\right)(8-5\lambda)}$$

Observe that $\left(4-\lambda\right)\left(11-8\lambda\right)-9(2-\lambda)=2\left(1-\lambda\right)\left(13-4\lambda\right)>0$

Also, we have $\left(\alpha_{h1}>\alpha_{l1}\right)$ whenever the high quality product is more expensive than the low quality product. It follows that $\beta_{h1}-\beta_{l1}>\frac{2\left(1-\lambda\right)\left(13-4\lambda\right)}{3\left(4-\lambda\right)(8-5\lambda)}>0$. Hence it follows that $\left(\beta_{h1}>\beta_{l1}\right)$.

Recall that $x=x_{u1}$ indicates the location of an unconstrained consumer indifferent between purchasing $\{H,L\}$, where $x_{u1}=\frac{\beta_{h1}-\beta_{l1}+t}{2t}$

Similarly, recall that $x=x_{c1}$ indicate the location of a cash-constrained consumer indifferent between purchasing L and not making a purchase, where $x_{c1}=\frac{t-\beta_{l1}}{t}$.

Unconstrained consumers located between $0<x<x_{u1}$ purchase H, while unconstrained consumers located between $x_{u1}<x<1$ purchase L. Cash-constrained consumers located between $\left(0<x<x_{c1}\right)$ do not make a purchase, while those consumers located between $\left(x_{c1}<x<1\right)$ purchase L. We can now measure the consumer surplus. The surplus of consumers who purchase H in the unconstrained segment is $CS_{hu1}=\lambda\int_0^{x_{u1}}U_{h1}^*(x)dx=\lambda\int_0^{x_{u1}}\left(\beta_{h1}-tx\right)dx$

This yields $CS_{hu1}=\lambda\left(\beta_{h1}x_{u1}-\frac{t}{2}x_{u1}^2\right)$.

Since no consumer in the cash-constrained segment can afford to purchase H, we have $CS_{hc1}=0$. Thus, the aggregate surplus of all consumers who purchase H is $CS_{h1}=CS_{hu1}+CS_{hc1}$, or $CS_{h1}=\lambda\left(\beta_{h1}x_{u1}-\frac{t}{2}{x_{u1}}^2\right)$.

Similarly, the surplus of consumers who purchase L in the unconstrained segment is $CS_{lu1} = \lambda \int_{x_{u1}}^{1} U_{l1}^{*}(x)dx = \lambda \int_{x_{u1}}^{1} \left(\beta_{l1} - t(1-x)\right)dx$.

This yields $CS_{lu1} = \lambda\left(\left(\beta_{l1} - t\right)(1-x_{u1}) + \frac{t}{2}\left(1-x_{u1}\right)^2\right)$

And the surplus of consumers who purchase L in the cash-constrained segment is

$CS_{lc1} = \left(1-\lambda\right)\int_{x_{c1}}^{1} U_{l1}(x)dx = \left(1-\lambda\right)\int_{x_{c1}}^{1} \left(\beta_{l1} - t(1-x)\right)dx$

This yields $CS_{lc1} = \left(1-\lambda\right)\left(\left(\beta_{l1} - t\right)(1-x_{c1}) + \frac{t}{2}\left(1-x_{c1}\right)^2\right)$.

Thus, the aggregate surplus of all consumers who purchase L is $CS_{l1} = CS_{lu1} + CS_{lc1}$, or

$$
\begin{aligned}
CS_{l1} &= \lambda\left(\left(\beta_{l1} - t\right)(1-x_{u1}) + \frac{t}{2}\left(1-x_{u1}\right)^2\right) + \left(1-\lambda\right)\left(\left(\beta_{l1} - t\right)(1-x_{c1}) + \frac{t}{2}\left(1-x_{c1}\right)^2\right) \\
&= \left(\beta_{l1} - t\right)\left(1-\lambda x_{u1} - \left(1-\lambda\right)x_{c1}\right) + \frac{t}{2}\left(\lambda\left(1-x_{u1}\right)^2 + \left(1-\lambda\right)\left(1-x_{c1}\right)^2\right) \\
&= \left(\beta_{l1} - t\right)\left(1-\lambda x_{u1} - \left(1-\lambda\right)x_{c1}\right) + \frac{t}{2}\left(1 + \lambda\left(-2x_{u1} + x_{u1}^2\right) + \left(1-\lambda\right)\left(-2x_{c1} + x_{c1}^2\right)\right) \\
&= \left(\beta_{l1} - t\right)\left(1-\lambda x_{u1} - \left(1-\lambda\right)x_{c1}\right) + \frac{t}{2} - t\left(\lambda x_{u1} + \left(1-\lambda\right)x_{c1}\right) + \frac{t}{2}\left(\lambda x_{u1}^2 + \left(1-\lambda\right)x_{c1}^2\right) \\
&= \beta_{l1}\left(1-\lambda x_{u1} - \left(1-\lambda\right)x_{c1}\right) - \frac{t}{2} + \frac{t}{2}\left(\lambda x_{u1}^2 + \left(1-\lambda\right)x_{c1}^2\right) \\
&= \beta_{l1}\left(1-\lambda x_{u1} - \left(1-\lambda\right)x_{c1}\right) - \frac{t}{2}\left(1-\lambda x_{u1}^2 - \left(1-\lambda\right)x_{c1}^2\right)
\end{aligned}
$$

Thus we get $CS_{l1} = \beta_{l1}\left(1-\lambda x_{u1} - \left(1-\lambda\right)x_{c1}\right) - \frac{t}{2}\left(1-\lambda x_{u1}^2 - \left(1-\lambda\right)x_{c1}^2\right)$

Finally, the aggregate consumer surplus of all consumers in the market, $CS_1 = CS_{h1} + CS_{l1}$ is

$$CS_1 = \lambda\left(\beta_{h1}x_{u1} - \frac{t}{2}x_{u1}^2\right) + \beta_{l1}\left(1-\lambda x_{u1} - \left(1-\lambda\right)x_{c1}\right) - \frac{t}{2}\left(1-\lambda x_{u1}^2 - \left(1-\lambda\right)x_{c1}^2\right)$$

This simplifies to $CS_1 = \beta_{h1}\lambda x_{u1} + \beta_{l1}\left(1-\lambda x_{u1} - \left(1-\lambda\right)x_{c1}\right) - \frac{t}{2}\left(1-\left(1-\lambda\right)x_{c1}^2\right)$

where $x_{u1} = \frac{t + \beta_{h1} - \beta_{l1}}{2t}$, $x_{c1} = \frac{t - \beta_{l1}}{t}$.

Appendix 3

Model of an Emerging Market with Unconstrained and Cash-Constrained Consumers: High-quality firm sells to both unconstrained and cash-constrained segments:

The firms' profit functions are

$$\Pi_{h2} = \left(\chi_c - k_h s_{h2}\right)\left(\frac{\beta_{h2} - \beta_{l2} + t}{2t}\right)$$

$$\Pi_{l2} = \left(p_{l2} - k_l s_{l2}\right)\left(1 - \frac{\beta_{h2} - \beta_{l2} + t}{2t}\right)$$

where $\beta_{h2} = \theta q_h f(s_{h2}) - \chi_c > 0$, $\beta_{l2} = \theta q_l f(s_{l2}) - p_{l2} > 0$.

For brevity, let us define α_{h2} and α_{l2} as $\alpha_{h2} = \theta q_h f(s_{h2}) - k_h s_{h2} > 0$, $\alpha_{l2} = \theta q_l f(s_{l2}) - k_l s_{l2} > 0$

In Stage 2 of this sub-game, both firms $\{H, L\}$ simultaneously choose their profit maximizing package sizes $\{s^*_{h2}, s^*_{l2}\}$. In Stage 3 of this sub-game, firm H sets its price as $p^*_{h2} = \chi_c$, while firm L chooses its profit maximizing price p^*_{l2}. We solve the game by backwards induction below, to determine the firms' equilibrium package sizes and prices.

Consider the Stage 3 pricing sub-game:

In Stage 3, firm L sets its equilibrium price such that its marginal benefit equals its marginal cost. The first-order and second-order conditions are $\frac{\partial \Pi_{l2}}{\partial p_{l2}} = 0$, $\frac{\partial^2 \Pi_{l2}}{\partial p^2_{l2}} < 0$. Solving the first-order condition, we find that in Stage 3, the firms set equilibrium prices $\{p^+_{h2}, p^+_{l2}\}$ as follows: $p^+_{h2} = \chi_c$ and

$$p^+_{l2} = \frac{\chi_c + t - \theta q_h f(s_{h2}) + \theta q_l f(s_{l2}) + k_l s_{l2}}{2}$$

$$= \frac{t - \beta_{h2} + \alpha_{l2}}{2} + k_l s_{l2}$$

Note that $\left(p^+_{l2} - k_l s_{l2}\right) = \frac{t - \beta_{h2} + \alpha_{l2}}{2}$ and $x_2 = \frac{t + \beta_{h2} - \beta_{l2}}{2t}$.

When we substitute the equilibrium price p^+_{l2} into the firms' profit functions, we get

$\Pi^+_{h2} = \frac{\left(\chi_c - k_h s_{h2}\right)\left(3t + \beta_{h2} - \alpha_{l2}\right)}{4t}$, $\Pi^+_{l2} = \frac{(t - \beta_{h2} + \alpha_{l2})^2}{8t}$,

where $\beta_{h2} = \theta q_h f(s_{h2}) - \chi_c > 0$, $\alpha_{l2} = \theta q_l f(s_{l2}) - k_l s_{l2} > 0$

Proceeding with backward induction, consider the Stage 2 package-sizing sub-game. In Stage 2, the firms set their equilibrium package sizes such that their marginal benefit equals their marginal cost. The first-order and second-order conditions are $\frac{\partial \Pi^+_{h2}}{\partial s_{h2}} = 0$, $\frac{\partial \Pi^+_{l2}}{\partial s_{l2}} = 0$, $\frac{\partial^2 \Pi^+_{h2}}{\partial s^2_{h2}} < 0$, $\frac{\partial^2 \Pi^+_{l2}}{\partial s^2_{l2}} < 0$. When we solve the first-order conditions, we find that in Stage 2 the firms set equilibrium package sizes $\{s^*_{h2}, s^*_{l2}\}$, as follows.

Solving $\frac{\partial \Pi^+_{l2}}{\partial s_{l2}} = 0$ yields $f'(s^*_{l2}) = \frac{k_l}{\theta q_l}$.

Now solving $\frac{\partial \Pi^+_{h2}}{\partial s_{h2}} = 0$ yields $\theta q_h f'(s^*_{h2})\left(\chi_c - k_h s^*_{h2}\right) = k_h\left(3t + \beta_{h2} - \alpha_{l2}\right)$.

We can now summarize the solution of the sub-game where H serves both segments:

In Stage 2, the firms set package sizes that satisfy $f'(s_{l2}^*) = \frac{k_l}{\theta q_l}$, $f'(s_{h2}^*) = \frac{k_h\left(3t + \beta_{h2} - \alpha_{l2}\right)}{\theta q_h\left(\chi_c - k_h s_{h2}^*\right)}$

In Stage 3, the firms sets prices that satisfy $p_{h2}^* = \chi_c$, $p_{l2}^* = \frac{t - \beta_{h2} + \alpha_{l2}}{2} + k_l s_{l2}^*$, where $\beta_{h2} = \theta q_h f(s_{h2}^*) - \chi_c > 0$, $\alpha_{l2} = \theta q_l f(s_{l2}^*) - k_l s_{l2}^* > 0$

The corresponding equilibrium margins, sales and profits of $\{H, L\}$ are as follows:

$$m_{h2}^* = \left(\chi_c - k_h s_{h2}\right) = \frac{k_h(3t + \alpha_{h2} - \alpha_{l2})}{\left(k_h + \theta q_h f'(s_{h2}^*)\right)}$$

$$d_{h2}^* = \frac{\left(3t + \beta_{h2} - \alpha_{l2}\right)}{4t} = \frac{\theta q_h(3t + \alpha_{h2} - \alpha_{l2}) f'(s_{h2}^*)}{4t\left(k_h + \theta q_h f'(s_{h2}^*)\right)}$$

$$\Pi_{h2}^* = \left(\chi_c - k_h s_{h2}\right)\frac{\left(3t + \beta_{h2} - \alpha_{l2}\right)}{4t} = \frac{\theta k_h q_h(3t + \alpha_{h2} - \alpha_{l2})^2 f'(s_{h2}^*)}{4t\left(k_h + \theta q_h f'(s_{h2}^*)\right)^2}$$

$$m_{l2}^* = \frac{t - \beta_{h2} + \alpha_{l2}}{2} = \frac{\left(4k_h t + \theta q_h(t - \alpha_{h2} + \alpha_{l2}) f'(s_{h2}^*)\right)}{2\left(k_h + \theta q_h f'(s_{h2}^*)\right)}$$

$$d_{l2}^* = \frac{t - \beta_{h2} + \alpha_{l2}}{4t} = \frac{\left(4k_h t + \theta q_h(t - \alpha_{h2} + \alpha_{l2}) f'(s_{h2}^*)\right)}{4t\left(k_h + \theta q_h f'(s_{h2}^*)\right)}$$

$$\Pi_{l2}^* = \frac{(t - \beta_{h2} + \alpha_{l2})^2}{8t} = \frac{\left(4k_h t + \theta q_h(t - \alpha_{h2} + \alpha_{l2}) f'(s_{h2}^*)\right)^2}{8t\left(k_h + \theta q_h f'(s_{h2}^*)\right)^2}$$

where $\alpha_{h2} = \theta q_h f(s_{h2}^*) - k_h s_{h2}^* > 0$, $\alpha_{l2} = \theta q_l f(s_{l2}^*) - k_l s_{l2}^* > 0$.

Next, we identify the necessary bounds on the unit transportation cost as $\alpha_{h2} - \alpha_{l2} < t < \beta_{h2} + \frac{\alpha_{l2}}{3}$.

Every consumer who purchases H should derive a strictly positive utility from her purchase. Let x_{a2} represent the location of a consumer indifferent between purchasing H and not purchasing. Then we must have $x_2 < x_{a2}$. Similarly, every consumer who purchases L should derive a strictly positive utility from her purchase. Let x_{b2} represent the location of a consumer indifferent between purchasing L and not purchasing. Then we must have $x_2 > x_{b2}$. Collectively, the necessary conditions for the market segments to be fully covered are $x_{b2} < x_2 < x_{a2}$. This inequality is satisfied if the following condition holds: $3t < 3\beta_{h2} + \alpha_{l2}$. Collectively, we see that the unit transportation cost is bounded as $\beta_{h2} - \alpha_{l2} < t < \beta_{h2} + \frac{\alpha_{l2}}{3}$, where $\beta_{h2} = \theta q_h f(s_{h2}) - \chi_c > 0$, $\alpha_{l2} = \theta q_l f(s_{l2}^*) - k_l s_{l2}^* > 0$.

Next, we establish the necessary conditions for the price of the high quality product to exceed the price of the low quality product. The high quality product is more expensive than the low quality product $(p_{h2}^* > p_{l2}^*)$, if $\left(\alpha_{h2} > \alpha_{l2}\right)$ and $\left(k_h s_{h2}^* > k_l s_{l2}^*\right)$. This is analogous to the benchmark model. The difference between the equilibrium prices of H and L is

$$p_{h2}^{*}-p_{l2}^{*}=\chi_c-p_{l2}^{*}$$

$$=\chi_c-\frac{t-\beta_{h2}+\alpha_{l2}}{2}-k_l s_{l2}$$

$$=\frac{2k_h t+\theta q_h f'(s_{h2}^{*})\left(\theta q_h f(s_{h2}^{*})-\theta q_l f(s_{l2}^{*})+k_h s_{h2}^{*}-k_l s_{l2}^{*}-t\right)+2\theta k_h\left(q_h f(s_{h2}^{*})-q_l f(s_{l2}^{*})\right)}{2\left(k_h+\theta q_h f'(s_{h2}^{*})\right)}$$

$$=\frac{\theta q_h f'(s_{h2}^{*})\left(\alpha_{h2}-\alpha_{l2}+2k_h s_{h2}^{*}-2k_l s_{l2}^{*}-t\right)+2k_h\left(\alpha_{h2}-\alpha_{l2}+k_h s_{h2}^{*}-k_l s_{l2}^{*}+t\right)}{2\left(k_h+\theta q_h f'(s_{h2}^{*})\right)}$$

$$=\frac{\left(\theta q_h f'(s_{h2}^{*})+2k_h\right)\left(\alpha_{h2}-\alpha_{l2}\right)+2\left(\theta q_h f'(s_{h2}^{*})+k_h\right)\left(k_h s_{h2}^{*}-k_l s_{l2}^{*}\right)+t\left(2k_h-\theta q_h f'(s_{h2}^{*})\right)}{2\left(k_h+\theta q_h f'(s_{h2}^{*})\right)}$$

The difference in equilibrium prices of H and L is

$$p_{h2}^{*}-p_{l2}^{*}=\frac{\left(\theta q_h f'(s_{h2}^{*})+2k_h\right)\left(\alpha_{h2}-\alpha_{l2}\right)+2\left(\theta q_h f'(s_{h2}^{*})+k_h\right)\left(k_h s_{h2}^{*}-k_l s_{l2}^{*}\right)+t\left(2k_h-\theta q_h f'(s_{h2}^{*})\right)}{2\left(k_h+\theta q_h f'(s_{h2}^{*})\right)}$$

From this, we see that the high quality product is more expensive than the low quality product $(p_{h2}^{*}>p_{l2}^{*})$, if $\left(\alpha_{h2}>\alpha_{l2}\right)$ and $\left(k_h s_{h2}^{*}>k_l s_{l2}^{*}\right)$

We now measure the consumer surplus. The surplus of consumers who purchase H in the unconstrained segment is $\lambda\int_0^{x_2}U_{h2}(x)dx$, while the surplus of consumers who purchase H in the cash-constrained segment is $\left(1-\lambda\right)\int_0^{x_2}U_{h2}(x)dx$. Thus, the aggregate consumer surplus of consumers who purchase H is $CS_{h2}=\int_0^{x_2}U_{h2}(x)dx=\int_0^{x_2}\left(\beta_{h2}-tx\right)dx$. This yields $CS_{h2}=\beta_{h2}x_2-\frac{t}{2}x_2^2$. Similarly, the surplus of consumers who purchase L in the unconstrained segment is $\lambda\int_{x_2}^{1}U_{l2}(x)dx$, while the surplus of consumers who purchase L in the cash-constrained segment is $\left(1-\lambda\right)\int_{x_2}^{1}U_{l2}(x)dx$. The aggregate consumer surplus of consumers who purchase L is $CS_{l2}=\int_{x_2}^{1}U_{l2}(x)dx=\int_{x_2}^{1}\left(\beta_{l2}-t(1-x)\right)dx$. This yields $CS_{l2}=\left(\beta_{l2}-t\right)(1-x_2)+\frac{t}{2}(1-x_2)^2$ where $x_2=\frac{\beta_{h2}-\beta_{l2}+t}{2t}$. The aggregate consumer surplus $CS_2=CS_{h2}+CS_{l2}$ is $CS_2=\beta_{h2}x_2-\frac{t}{2}x_2^2+\left(\beta_{l2}-t\right)(1-x_2)+\frac{t}{2}(1-x_2)^2$

Appendix 4:

Proof of Proposition 1: When the high quality firm sells exclusively to the unconstrained segment in the emerging market, each firm sets the same package size in the emerging market, as compared to its package size in the developed market, i.e. $s_{h1}^{*}=s_h^{*}$ and $s_{l1}^{*}=s_l^{*}$.

We prove $\left(s_{h1}^{*}=s_{h}^{*}\right)$ by comparing the first-order conditions of the high quality firm's profit maximization problem. In Appendix 1, we show that when both market segments are unconstrained, $\frac{\partial \Pi_h}{\partial s_h}=\frac{\left(3t+\theta q_h f(s_h)-k_h s_h-\theta q_l f(s_l)+k_l s_l\right)\left(\theta q_h f'(s_h)-k_h\right)}{9t}$

We also show that $\left(3t+\theta q_h f(s_h)-k_h s_h-\theta q_l f(s_l)+k_l s_l\right)>0$ and thus the first order condition yields $\left(\theta q_h f'(s_h)-k_h\right)=0$.

Similarly, in Appendix 2, we show that when one market segment is cash-constrained, while the other segment is unconstrained, solving $\frac{\partial \Pi_{h1}}{\partial s_{h1}}=0$ yields $\left(\theta q_h f'(s_{h1})-k_h\right)=0$.

On comparison, we see that $f'(s_h)=f'(s_{h1})=\frac{k_h}{\theta q_h}$ and therefore, $\left(s_{h1}^{*}=s_{h}^{*}\right)$.

We prove $\left(s_{l1}^{*}=s_{l}^{*}\right)$ similarly by comparing the first-order conditions of the low quality firm's profit maximization problem. In Appendix 1, we show that when both market segments are unconstrained,

$$\frac{\partial \Pi_l}{\partial s_l}=\frac{\left(3t-\theta q_h f(s_h)+k_h s_h+\theta q_l f(s_l)-k_l s_l\right)\left(\theta q_l f'(s_l)-k_l\right)}{9t}=0$$

We also show that $\left(3t-\theta q_h f(s_h)+k_h s_h+\theta q_l f(s_l)-k_l s_l\right)>0$ and thus $\left(\theta q_l f'(s_l)-k_l\right)=0$.

Similarly, in Appendix 2, we show that when one market segment is cash-constrained, while the other segment is unconstrained, solving $\frac{\partial \Pi_{l1}}{\partial s_{l1}}=0$ yields $\left(\theta q_l f'(s_{l1})-k_l\right)=0$.

On comparison, we see that $f'(s_l)=f'(s_{l1})=\frac{k_l}{\theta q_l}$ and therefore, $\left(s_{l1}^{*}=s_{l}^{*}\right)$. QED

Proof of Proposition 2: *When the high quality firm sells exclusively to the unconstrained segment in the emerging market, each firm sets a higher price in the emerging market, as compared to its price in the developed market, i.e.* $p_{h1}^{*}>p_{h}^{*}$ *and* $p_{l1}^{*}>p_{l}^{*}$.

The equilibrium prices set by the high quality firm in the benchmark case is $p_h^{*}=\frac{3t+\alpha_h-\alpha_l}{3}+k_h s_h^{*}$, where $\alpha_h=\theta q_h f(s_h^{*})-k_h s_h^{*}>0$, $\alpha_l=\theta q_l f(s_l^{*})-k_l s_l^{*}>0$.

The equilibrium prices set by the high quality firm when the high quality firm does not sell to the cash-constrained market segment is $p_{h1}^{*}=\frac{((4-\lambda)t+(4-3\lambda)\alpha_{h1}-(2-\lambda)\alpha_{l1})}{(8-5\lambda)}+k_h s_{h1}^{*}$, where $\alpha_{h1}=\theta q_h f(s_{h1}^{*})-k_h s_{h1}^{*}>0$, $\alpha_{l1}=\theta q_l f(s_{l1}^{*})-k_l s_{l1}^{*}>0$.

From Proposition 1, $s_{h1}^{*}=s_{h}^{*}$. Also, $\alpha_{h1}=\alpha_h$ and $\alpha_{l1}=\alpha_l$

Substituting and comparing the prices, we have

$$p_{h1}^* - p_h^* = \left(\frac{(4-\lambda)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}}{(8-5\lambda)} + k_h s_{h1}^* \right) - \left(\frac{3t + \alpha_h - \alpha_l}{3} + k_h s_h^* \right)$$

$$= \left(\frac{(4-\lambda)t + (4-3\lambda)\alpha_h - (2-\lambda)\alpha_l}{(8-5\lambda)} \right) - \left(\frac{3t + \alpha_h - \alpha_l}{3} \right)$$

Simplifying, we get $p_{h1}^* > p_h^*$ if and only if $2\alpha_h + \alpha_l - 6t > 0$, or $t < \frac{2\alpha_h + \alpha_l}{6}$.

Since the unit transportation cost is bounded as $\frac{\alpha_h - \alpha_l}{3} < t < \frac{2\alpha_h + \alpha_l}{6}$, we conclude that $p_{h1}^* > p_h^*$. Similarly, consider the equilibrium prices set by the low quality firm in the benchmark case and when the high quality firm does not sell to the cash-constrained market segment: $p_l^* = \frac{3t - \alpha_h + \alpha_l}{3} + k_l s_l^*$, $p_{l1}^* = \frac{(3\lambda t - \lambda\alpha_{h1} + (4-3\lambda)\alpha_{l1})}{(8-5\lambda)} + k_l s_{l1}^*$.

Substituting $s_{h1}^* = s_h^*$, $\alpha_{h1} = \alpha_h$, $\alpha_{l1} = \alpha_l$

$$p_{l1}^* - p_l^* = \left(\frac{(3\lambda t - \lambda\alpha_{h1} + (4-3\lambda)\alpha_{l1})}{(8-5\lambda)} + k_l s_{l1}^* \right) - \left(\frac{3t - \alpha_h + \alpha_l}{3} + k_l s_l^* \right)$$

$$= \frac{3\lambda t - \lambda\alpha_h + (4-3\lambda)\alpha_l}{(8-5\lambda)} - \frac{3t - \alpha_h + \alpha_l}{3}$$

$$= \frac{3(3\lambda t - \lambda\alpha_h + (4-3\lambda)\alpha_l) - (8-5\lambda)(3t - \alpha_h + \alpha_l)}{3(8-5\lambda)}$$

$$= \frac{-24(1-\lambda)t + 8\alpha_h(1-\lambda) + 4\alpha_l(1-\lambda)}{3(8-5\lambda)}$$

$$= \frac{4(1-\lambda)(2\alpha_h + \alpha_l - 6t)}{3(8-5\lambda)}$$

Simplifying, we get $p_{l1}^* > p_l^*$ if and only if $2\alpha_h + \alpha_l - 6t > 0$, or $t < \frac{2\alpha_h + \alpha_l}{6}$. Since the unit transportation cost is bounded as $\frac{\alpha_h - \alpha_l}{3} < t < \frac{2\alpha_h + \alpha_l}{6}$ we conclude that $p_{l1}^* > p_l^*$.

QED

Proof of Proposition 3: *When the high quality firm sells exclusively to the unconstrained segment in the emerging market, the low-quality firm makes a higher profit in the emerging market, as compared to its profit in the developed market, i.e.* $\Pi_{l1}^* > \Pi_l^*$.

First we prove the following result:

The locations of indifferent consumers on the Hotelling lines are related as $(x_{c1} < x_0 < x_{u1})$.

We have $x_0 = \frac{1}{2} + \frac{\alpha_h - \alpha_l}{6t}$, $x_{u1} = \frac{\beta_{h1} - \beta_{l1} + t}{2t}$, $x_{c1} = \frac{t - \beta_{l1}}{t}$, where

$\beta_{h1} = \frac{2(2-\lambda)\alpha_{h1} + (2-\lambda)\alpha_{l1} - (4-\lambda)t}{(8-5\lambda)}$, $\beta_{l1} = \frac{\lambda\alpha_{h1} + 2(2-\lambda)\alpha_{l1} - 3\lambda t}{(8-5\lambda)}$, $\alpha_{h1} = \alpha_h$, $\alpha_{l1} = \alpha_l$

This yields

$$\begin{aligned}
x_0 - x_{c1} &= \frac{1}{2} + \frac{\alpha_h - \alpha_l}{6t} - \frac{t - \beta_{l1}}{t} \\
&= \frac{\alpha_h - \alpha_l - 3t}{6t} + \frac{\beta_{l1}}{t} \\
&= \frac{\alpha_h - \alpha_l - 3t}{6t} + \frac{\lambda\alpha_h + 2(2-\lambda)\alpha_l - 3\lambda t}{(8-5\lambda)t} \\
&= \frac{(8-5\lambda)\left(\alpha_h - \alpha_l - 3t\right) + 6\left(\lambda\alpha_h + 2(2-\lambda)\alpha_l - 3\lambda t\right)}{6(8-5\lambda)t} \\
&= \frac{(8+\lambda)\alpha_h + \left(16-7\lambda\right)\alpha_l - 3\left(8+\lambda\right)t}{6(8-5\lambda)t}
\end{aligned}$$

Since the unit transportation cost is bounded as $\frac{\alpha_h - \alpha_l}{3} < t < \frac{2\alpha_h + \alpha_l}{6}$,

$$\begin{aligned}
x_0 - x_{c1} &> \frac{2(8+\lambda)\alpha_h + 2\left(16-7\lambda\right)\alpha_l - \left(8+\lambda\right)\left(2\alpha_h + \alpha_l\right)}{(8-5\lambda)\left(2\alpha_h + \alpha_l\right)} \\
&= \frac{3\alpha_l}{2\left(\alpha_h + \alpha_l\right)} \\
&> 0
\end{aligned}$$

Thus we have $\left(x_0 - x_{c1} > 0\right)$. Similarly,

$$x_0 - x_{u1} = \left(\frac{1}{2} + \frac{\alpha_h - \alpha_l}{6t}\right) - \left(\frac{\beta_{h1} - \beta_{l1} + t}{2t}\right)$$
$$= \frac{(3t + \alpha_h - \alpha_l) - 3(\beta_{h1} - \beta_{l1} + t)}{6t}$$
$$= \frac{(\alpha_h - \alpha_l) - 3(\beta_{h1} - \beta_{l1})}{6t}$$
$$= \frac{(\alpha_h - \alpha_l)}{6t} - \frac{(\beta_{h1} - \beta_{l1})}{2t}$$
$$= \frac{(\alpha_h - \alpha_l)}{6t} - \frac{(4-3\lambda)\alpha_h - (2-\lambda)\alpha_l - 4(1-\lambda)t}{2t(8-5\lambda)}$$
$$= \frac{(8-5\lambda)(\alpha_h - \alpha_l) - 3(4-3\lambda)\alpha_h + 3(2-\lambda)\alpha_l + 12(1-\lambda)t}{6t(8-5\lambda)}$$
$$= \frac{(8-5\lambda-3(4-3\lambda))\alpha_h + 3(2-\lambda)\alpha_l - (8-5\lambda)\alpha_l + 12(1-\lambda)t}{6t(8-5\lambda)}$$
$$= \frac{-4(1-\lambda)\alpha_h - 2(1-\lambda)\alpha_l + 12(1-\lambda)t}{6t(8-5\lambda)}$$
$$= \frac{(1-\lambda)(6t - 2\alpha_h - \alpha_l)}{3t(8-5\lambda)}$$

Thus we get $x_0 - x_{u1} = \dfrac{(1-\lambda)(6t - 2\alpha_h - \alpha_l)}{2(8-5\lambda)(\alpha_h + \alpha_l)}$.

Since the unit transportation cost is bounded as $\dfrac{\alpha_h - \alpha_l}{3} < t < \dfrac{2\alpha_h + \alpha_l}{6}$, we have $(6t - 2\alpha_h - \alpha_l) < 0$, which implies that $(x_0 - x_{u1}) < 0$. Combining $(x_0 - x_{c1} > 0)$ and $(x_0 - x_{u1}) < 0$, we get $(x_{c1} < x_0 < x_{u1})$. Now consider the margin of the low quality product. The equilibrium margin of the low quality firm relatively increases $(m_{l1}^* > m_l^*)$, since its equilibrium price is higher $(p_{l1}^* > p_l^*)$ and its package size is unchanged $(s_{h1}^* = s_h^*)$. The low quality firm's sales are also relatively higher $(d_{l1}^* > d_l^*)$. To see this, note from Appendix 1 and 2 that the sales of the low quality firm in the benchmark case and when the high quality firm does not sell to the cash-constrained market segment are given as follows:

$$d_{l1}^* = \frac{(2-\lambda)(3\lambda t - \lambda\alpha_{h1} + (4-3\lambda)\alpha_{l1})}{2t(8-5\lambda)},\ d_l^* = \frac{1}{2} - \frac{\alpha_h - \alpha_l}{6t},\ \begin{matrix}\alpha_{h1} = \alpha_h \\ \alpha_{l1} = \alpha_l\end{matrix}$$

$$d_{l1}^* - d_l^* = \frac{(2-\lambda)\left(3\lambda t - \lambda\alpha_h + (4-3\lambda)\alpha_l\right)}{2t(8-5\lambda)} - \frac{3t-\alpha_h+\alpha_l}{6t}$$

$$= \frac{3(2-\lambda)\left(3\lambda t - \lambda\alpha_h + (4-3\lambda)\alpha_l\right) - (8-5\lambda)\left(3t-\alpha_h+\alpha_l\right)}{6t(8-5\lambda)}$$

$$= \frac{\left[9\lambda(2-\lambda) - 3(8-5\lambda)\right]t + \left[(8-5\lambda) - 3\lambda(2-\lambda)\right]\alpha_h + \left[3(2-\lambda)(4-3\lambda) - (8-5\lambda)\right]\alpha_l}{6t(8-5\lambda)}$$

$$= \frac{-\left(24-33\lambda+9\lambda^2\right)t + \left(8-11\lambda+3\lambda^2\right)\alpha_h + \left(16-25\lambda+9\lambda^2\right)\alpha_l}{6t(8-5\lambda)}$$

$$= \frac{(1-\lambda)\left((24-9\lambda)\alpha_h + (16-9\lambda)\alpha_l - 3(8-3\lambda)t\right)}{6t(8-5\lambda)}$$

Since $\frac{\alpha_h-\alpha_l}{3} < t < \frac{2\alpha_h+\alpha_l}{6}$,

$$d_{l1}^* - d_l^* > \frac{(1-\lambda)\left(2(24-9\lambda)\alpha_h + 2(16-9\lambda)\alpha_l - (8-3\lambda)(2\alpha_h+\alpha_l)\right)}{2(8-5\lambda)(2\alpha_h+\alpha_l)}$$

$$= \frac{(1-\lambda)\left(\left(2(24-9\lambda) - 2(8-3\lambda)\right)\alpha_h + \left(2(16-9\lambda) - (8-3\lambda)\right)\alpha_l\right)}{2(8-5\lambda)(2\alpha_h+\alpha_l)}$$

$$= \frac{(1-\lambda)\left((32-12\lambda)\alpha_h + (24-15\lambda)\alpha_l\right)}{2(8-5\lambda)(2\alpha_h+\alpha_l)}$$

$$= \frac{(1-\lambda)\left(4(8-3\lambda)\alpha_h + 3(8-5\lambda)\alpha_l\right)}{2(8-5\lambda)(2\alpha_h+\alpha_l)}$$

Now observe that $\left(4(8-3\lambda)\alpha_h + 3(8-5\lambda)\alpha_l\right) > 0$. Thus, it follows that $d_{l1}^* - d_l^* > 0$.

Since both the margin as well as the sales of the low quality firm are relatively higher, it follows that the low quality firm makes a relatively larger profit $\left(\Pi_{l1}^* > \Pi_l^*\right)$.

QED

Proof of Proposition 4: *When the high-quality firm sells to both segments in the emerging market, the high quality firm sets a smaller package size in the emerging market, as compared to its package size in the developed market, i.e.* $s_{h2}^* < s_h^*$. *In contrast, the low quality firm sets the same package size in both markets, i.e.* $s_{l2}^* = s_l^*$.

We can derive $\left(s_{h2}^* < s_h^*\right)$ by applying the Implicit Function Theorem.

We prove $\left(s_{l2}^* = s_l^*\right)$ by comparing the first-order conditions of the firm's profit maximization problem. In Appendix 1, we show that when both market segments are unconstrained,

$$\frac{\partial \Pi_l}{\partial s_l} = \frac{\left(3t - \theta q_h f(s_h) + k_h s_h + \theta q_l f(s_l) - k_l s_l\right)\left(\theta q_l f'(s_l) - k_l\right)}{9t} = 0$$

We also show that $\left(3t - \theta q_h f(s_h) + k_h s_h + \theta q_l f(s_l) - k_l s_l\right) > 0$ and thus $\left(\theta q_l f'(s_l) - k_l\right) = 0$. Similarly, in Appendix 3, we show that when one market segment is cash-constrained, while the other segment is unconstrained, solving $\frac{\partial \Pi_{l2}}{\partial s_{l2}} = 0$ yields $\left(\theta q_l f'(s_{l2}) - k_l\right) = 0$.

On comparison, we see that $f'(s_l) = f'(s_{l2}) = \frac{k_l}{\theta q_l}$

and therefore, $\left(s_{l2}^* = s_l^*\right)$.

QED

Proof of Proposition 5: *The low quality firm raises its price* $\left(p_{l2}^* > p_l^*\right)$ *when the high quality firm sells to both the cash-constrained and unconstrained segments, as compared to the benchmark case.*

Before proving this proposition, we will prove the following result:
The base value offered by the high quality firm declines when it lowers its price to match the cash constraint of consumers, as compared to the benchmark case, $\left(\beta_{h2} < \beta_h\right)$ or $\left(\theta q_h f(s_{h2}^*) - \chi_c\right) < \left(\theta q_h f(s_h^*) - p_h\right)$

From our earlier analysis, we know that

$$\beta_h = \left(\theta q_h f(s_h^*) - p_h\right) = \frac{2\alpha_h + \alpha_l}{3} - t$$

$$\beta_{h2} = \left(\theta q_h f(s_{h2}^*) - \chi_c\right) = \frac{\left(\alpha_{h2}\theta q_h f'(s_{h2}^*) - k_h\left(3t - \alpha_l\right)\right)}{\left(k_h + \theta q_h f'(s_{h2}^*)\right)}$$

where $\alpha_h = \theta q_h f(s_h^*) - k_h s_h^* > 0$, $\alpha_l = \theta q_l f(s_l^*) - k_l s_l^* > 0$, $\alpha_{h2} = \theta q_h f(s_{h2}^*) - k_h s_{h2}^* > 0$, $\alpha_{l2} = \theta q_l f(s_{l2}^*) - k_l s_{l2}^* > 0$. We also know from Proposition 4 that the low quality firm sets the same package size, or $\left(s_{l2}^* = s_l^*\right)$, implying that $\left(\alpha_{l2} = \alpha_l\right)$. We have

$$\beta_h - \beta_{h2} = \frac{\left(2\alpha_h + \alpha_l - 3t\right)}{3} - \frac{\left(\alpha_{h2}\theta q_h f'(s_{h2}^*) - k_h\left(3t - \alpha_l\right)\right)}{\left(k_h + \theta q_h f'(s_{h2}^*)\right)}$$

Define the following function in s_{h2}

$$G(s_{h2}) = \frac{\left(\alpha_{h2}\theta q_h f'(s_{h2}) - k_h\left(3t - \alpha_l\right)\right)}{\left(k_h + \theta q_h f'(s_{h2})\right)}$$

where $\alpha_{h2} = \theta q_h f(s_{h2}) - k_h s_{h2}$, yielding

$$G(s_{h2}) = \frac{\theta q_h f'(s_{h2})\left(\theta q_h f(s_{h2}) - k_h s_{h2}\right) - k_h\left(3t - \alpha_l\right)}{\left(k_h + \theta q_h f'(s_{h2})\right)}$$

Notice that we have $\beta_h - \beta_{h2} = \frac{\left(2\alpha_h + \alpha_l - 3t\right)}{3} - G(s_{h2}^*)$

We will now prove that $\beta_{h2} = G(s_{h2}^*) < G(s_h^*)$. From Proposition 4, we know that the high quality firm reduces its equilibrium package size $\left(s_{h2}^* < s_h^*\right)$ when it lowers its price to sell to the cash-constrained segment. If $\left(s_{h2} < s_h\right)$, then we will have $G(s_{h2}) < G(s_h)$ if and only if, the function $G(s_{h2})$ is increasing in s_{h2}, or $\frac{\partial G(s_{h2})}{\partial s_{h2}} > 0$. We will prove that $G(s_{h2})$ is increasing in s_{h2}, where $G(s_{h2}) = \frac{\theta q_h f'(s_{h2})\left(\theta q_h f(s_{h2}) - k_h s_{h2}\right) - k_h\left(3t - \alpha_l\right)}{\left(k_h + \theta q_h f'(s_{h2})\right)}$.

We will have $\frac{\partial G(s_{h2})}{\partial s_{h2}} > 0$ if and only if

$$\begin{aligned}
&\left(k_h + \theta q_h f'(s_{h2})\right)\frac{\partial}{\partial s_{h2}}\left(\theta q_h f'(s_{h2})\left(\theta q_h f(s_{h2}) - k_h s_{h2}\right) - k_h\left(3t - \alpha_l\right)\right) \\
&-\left(\theta q_h f'(s_{h2})\left(\theta q_h f(s_{h2}) - k_h s_{h2}\right) - k_h\left(3t - \alpha_l\right)\right)\frac{\partial}{\partial s_{h2}}\left(k_h + \theta q_h f'(s_{h2})\right) \\
&> 0
\end{aligned}$$

Differentiating with respect to s_{h2} and simplifying, the above inequality becomes

$$\begin{aligned}
&\theta q_h f'(s_{h2}) * \left(\theta q_h f'(s_{h2}) + k_h\right) * \left(\theta q_h f'(s_{h2}) - k_h\right) \\
&-\theta q_h f''(s_{h2}) k_h \left(\alpha_h + \alpha_l - 3t\right) \\
&> 0
\end{aligned}$$

We know that $f'(s_h^*) = \frac{k_h}{\theta q_h}$ or $\theta q_h f'(s_{h2}) = k_h$.

We also know that $s_{h2} < s_h => f'(s_{h2}) > f'(s_h)$. Thus, we have $\left(\theta q_h f'(s_{h2}) - k_h\right) > 0$, implying that the first term is positive:

$\theta q_h f'(s_{h2}) * \left(\theta q_h f'(s_{h2}) + k_h\right) * \left(\theta q_h f'(s_{h2}) - k_h\right) > 0$

We know that $\frac{\alpha_h - \alpha_l}{3} < t < \frac{\alpha_h + \alpha_l}{3}$ or $\left(\alpha_h + \alpha_l - 3t\right) > 0$.

Since $f(s)$ is a concave function, $f''(s_{h2}) < 0$, implying that the second term is negative:

$\theta q_h f''(s_{h2}) k_h \left(\alpha_h + \alpha_l - 3t\right) < 0$

Collectively, it follows that

$$\begin{aligned}
&\theta q_h f'(s_{h2}) * \left(\theta q_h f'(s_{h2}) + k_h\right) * \left(\theta q_h f'(s_{h2}) - k_h\right) \\
&-\theta q_h f''(s_{h2}) k_h \left(\alpha_h + \alpha_l - 3t\right) \\
&> 0
\end{aligned}$$

or $\frac{\partial G(s_{h2})}{\partial s_{h2}} > 0$, implying that $G(s_{h2}) < G(s_h)$.

Thus, it follows that $\beta_{h2} = G(s_{h2}^*) < G(s_h^*)$, implying that $\beta_h - \beta_{h2} > \frac{(2\alpha_h + \alpha_l - 3t)}{3} - G(s_h^*)$, or

$$\beta_h - \beta_{h2} > \frac{(2\alpha_h + \alpha_l - 3t)}{3} - \frac{(\alpha_h \theta q_h f'(s_h^*) - k_h(3t - \alpha_l))}{(k_h + \theta q_h f'(s_h^*))}$$

Now recall that when the market only has unconstrained customers, H sets its equilibrium package size s_h^* such that $(\theta q_h f'(s_h^*) - k_h) = 0$ or $f'(s_h^*) = \frac{k_h}{\theta q_h}$.

Substituting $f'(s_h^*)$, on the right hand side, we get

$$\beta_h - \beta_{h2} > \frac{(2\alpha_h + \alpha_l - 3t)}{3} - \frac{(\alpha_h k_h - k_h(3t - \alpha_l))}{(k_h + k_h)}$$
$$= \frac{(2\alpha_h + \alpha_l - 3t)}{3} - \frac{(\alpha_h - (3t - \alpha_l))}{2}$$
$$= \frac{(\alpha_h - \alpha_l + 3t)}{6}$$

Since $\alpha_h > \alpha_l$ we get $\beta_h - \beta_{h2} > 0$, or $(\beta_{h2} < \beta_h)$, or $(\theta q_h f(s_{h2}^*) - \chi_c) < (\theta q_h f(s_h^*) - p_h)$

Let us now compare the pricing reaction functions.

When both market segments are unconstrained, the profit function of firm L is

$$\Pi_l = (p_l - k_l s_l)\left(1 - \frac{\theta q_h f(s_h) - p_h - \theta q_l f(s_l) + p_l + t}{2t}\right)$$

Firm L sets a price such that its marginal revenue equals its marginal cost, or

$$\frac{\partial \Pi_l}{\partial p_l} = \frac{p_h - 2p_l + k_l s_l + t - \theta(q_h f(s_h) - q_l f(s_l))}{2t} = 0$$

This yields the following pricing reaction function for L:

$$p_l = \frac{p_h + k_l s_l + t - \theta q_h f(s_h) + \theta q_l f(s_l)}{2}$$

In contrast, when one segment is cash constrained and the other segment is unconstrained and the high quality firm lowers its price in order to sell to both segments, the profit function of L is

$$\Pi_{l2} = (p_{l2} - k_l s_{l2})\left(1 - \frac{\theta q_h f(s_{h2}) - \chi - \theta q_l f(s_{l2}) + p_{l2} + t}{2t}\right).$$

We have $\frac{\partial \Pi_{l2}}{\partial p_{l2}} = \frac{\chi_c - 2p_{l2} + k_l s_{l2} + t - \theta q_h f(s_{h2}) + \theta q_l f(s_{l2})}{2t} = 0$

This yields the following pricing reaction function for L:

$$p_{l2} = \frac{\chi_c + t - \theta q_h f(s_{h2}) + \theta q_l f(s_{l2}) + k_l s_{l2}}{2}$$

Let us compare the two pricing reaction functions.

From Proposition 4, we know that firm L sets the same package size, or $s_{l2} = s_l$. We have

$$p_{l2} - p_l = \frac{\chi_c + t - \theta q_h f(s_{h2}) + \theta q_l f(s_{l2}) + k_l s_{l2}}{2} - \frac{p_h + k_l s_l + t - \theta q_h f(s_h) + \theta q_l f(s_l)}{2}$$
$$= \frac{\chi_c - \theta q_h f(s_{h2}) - p_h + \theta q_h f(s_h)}{2}$$
$$= \frac{\beta_h - \beta_{h2}}{2}$$

where $\beta_h = \theta q_h f(s_h) - p_h > 0$, $\beta_{h2} = \theta q_h f(s_{h2}) - \chi_c > 0$. This implies that we have $p_{l2} - p_l > 0$, if and only if $\beta_h - \beta_{h2} > 0$. However, from above, we know that $\left(\beta_{h2} < \beta_h\right)$. Hence it follows that $p_{l2} - p_l > 0$, or that L sets a relatively higher price.
QED

Proof of Proposition 6: *When the high-quality firm sells to both segments in the emerging market, the low-quality firm earns a higher profit in the emerging market, as compared to its profit in the developed market, i.e.* $\Pi_{l2}^* > \Pi_l^*$.

Before proving this proposition, we will prove the following result:
The locations of indifferent consumers on the Hotelling lines are related as $\left(x_2 < x_0\right)$

We have $x_0 = \frac{1}{2} + \frac{\alpha_h - \alpha_l}{6t}$, $x_2 = \frac{(3t + \alpha_{h2} - \alpha_{l2})\theta q_h f'(s_{h2}^*)}{4t\left(k_h + \theta q_h f'(s_{h2}^*)\right)}$.

Recall that $\alpha_{l2} = \alpha_l$. We have $x_0 - x_2 = \frac{3t + \alpha_h - \alpha_l}{6t} - \frac{(3t + \alpha_{h2} - \alpha_l)\theta q_h f'(s_{h2}^*)}{4t\left(k_h + \theta q_h f'(s_{h2}^*)\right)}$

Define the following function in s_{h2}: $G(s_{h2}) = \frac{(3t + \alpha_{h2} - \alpha_l)\theta q_h f'(s_{h2})}{4t\left(k_h + \theta q_h f'(s_{h2})\right)}$, where $\alpha_{h2} = \theta q_h f(s_{h2}) - k_h s_{h2}$, yielding $G(s_{h2}) = \frac{(3t - \alpha_l + \theta q_h f(s_{h2}) - k_h s_{h2})\theta q_h f'(s_{h2})}{4t\left(k_h + \theta q_h f'(s_{h2})\right)}$.

Notice that we have $x_0 - x_2 = \frac{3t + \alpha_h - \alpha_l}{6t} - G(s_{h2}^*)$.

We will now prove that $x_2 = G(s_{h2}^*) < G(s_h^*)$. From Proposition 4 , we know that the high quality firm reduces its equilibrium package size $\left(s_{h2}^* < s_h^*\right)$ when it lowers its price to sell to the cash-constrained segment. If $\left(s_{h2} < s_h\right)$, then we will have $G(s_{h2}) < G(s_h)$ if and only if, the function $G(s_{h2})$ is increasing in s_{h2}, or $\frac{\partial G(s_{h2})}{\partial s_{h2}} > 0$. We will prove that $G(s_{h2})$ is increasing in s_{h2}, where $G(s_{h2}) = \frac{(3t - \alpha_l + \theta q_h f(s_{h2}) - k_h s_{h2})\theta q_h f'(s_{h2})}{4t\left(k_h + \theta q_h f'(s_{h2})\right)}$. We will have $\frac{\partial G(s_{h2})}{\partial s_{h2}} > 0$ if and only if

$$\left(k_h+\theta q_h f'(s_{h2})\right)\frac{\partial}{\partial s_{h2}}\left((3t-\alpha_l+\theta q_h f(s_{h2})-k_h s_{h2})\theta q_h f'(s_{h2})\right)$$
$$-\left(\theta q_h f'(s_{h2})\right)\left((3t-\alpha_l+\theta q_h f(s_{h2})-k_h s_{h2})\theta q_h f'(s_{h2})\right)$$
$$>0$$

Differentiating with respect to s_{h2} and simplifying, the above inequality becomes

$$\left(\theta q_h f'(s_{h2})-k_h\right)\left((3t-\alpha_l+\theta q_h f(s_{h2})-k_h s_{h2})\theta q_h f''(s_{h2})+\theta q_h f'(s_{h2})(\theta q_h f'(s_{h2})-k_h)\right)$$
$$-\theta q_h f''(s_{h2})k_h\left(\alpha_h+\alpha_l-3t\right)$$
$$>0$$

This simplifies to

$$\left(\theta q_h f'(s_{h2})-k_h\right)(3t-\alpha_l+\theta q_h f(s_{h2})-k_h s_{h2})\theta q_h f''(s_{h2})$$
$$+\left(\theta q_h f'(s_{h2})-k_h\right)\theta q_h f'(s_{h2})(\theta q_h f'(s_{h2})-k_h)$$
$$-k_h\left(\alpha_h+\alpha_l-3t\right)\theta q_h f''(s_{h2})$$
$$>0$$

$$\theta q_h f'(s_{h2})(\alpha_h+\alpha_l-3t)\theta q_h f''(s_{h2})$$
$$+\theta q_h f'(s_{h2})(\theta q_h f'(s_{h2})-k_h)^2$$
$$>0$$

From the solution of Proposition 2, $f'(s_h^*)=\frac{k_h}{\theta q_h}$ or $\theta q_h f'(s_{h2})=k_h$. We also know that $s_{h2}<s_h \Rightarrow f'(s_{h2})>f'(s_h)$. Thus, we have $\left(\theta q_h f'(s_{h2})-k_h\right)>0$. This implies that the second term is positive, or $\theta q_h f'(s_{h2})(\theta q_h f'(s_{h2})-k_h)^2>0$. Now consider the first term $\theta q_h f'(s_{h2})(\alpha_h+\alpha_l-3t)\theta q_h f''(s_{h2})$. Since $\frac{\alpha_h-\alpha_l}{3}<t<\frac{2\alpha_h+\alpha_l}{6}$, we have $(\alpha_h+\alpha_l-3t)<0$ Since $f''(s_{h2})<0$, it follows that the first term is also positive, or $\theta q_h f'(s_{h2})(\alpha_h+\alpha_l-3t)\theta q_h f''(s_{h2})>0$. Hence it follows that $\frac{\partial G(s_{h2})}{\partial s_{h2}}>0$, implying that $G(s_{h2})<G(s_h)$. Thus, it follows that $x_2=G(s_{h2}^*)<G(s_h^*)$, implying that

$x_0-x_2>\frac{3t+\alpha_h-\alpha_l}{6t}-G(s_h^*)$, $x_0-x_2>\frac{3t+\alpha_h-\alpha_l}{6t}-\frac{(3t-\alpha_l+\theta q_h f(s_h^*)-k_h s_{h2})\theta q_h f'(s_h^*)}{4t\left(k_h+\theta q_h f'(s_h^*)\right)}$

Since $\left(\theta q_h f'(s_h^*)-k_h\right)=0$, $x_0-x_2>\frac{3t+\alpha_h-\alpha_l}{6t}-\frac{3t-\alpha_l+\alpha_h}{8t}$, or $x_0-x_2>\frac{\left(3t+\alpha_h-\alpha_l\right)}{24t}>0$

Hence it follows that $x_0-x_2>0$, or $x_0>x_2$. Now consider the sales of L, given that $x_0>x_2$. The sales of L are $d_{l2}=\left(1-x_2\right)$ and $d_l=\left(1-x_0\right)$. Since $\left(1-x_2\right)>\left(1-x_0\right)$, it follows that the sales of L increase when the high quality firm sells to both the constrained and unconstrained segments, as compared to the benchmark case. From Proposition 5, the price of L increases $\left(p_{l2}^*>p_l^*\right)$. Its marginal cost increases since from Proposition 4, its package size is the same $\left(s_{l2}^*>s_l^*\right)$. Hence it follows that the margin of L also increases when the high quality firm sells to both the constrained and unconstrained segments, as compared to the benchmark case. It collectively follows that it makes a relatively larger profit $\left(\Pi_{l2}^{*} > \Pi_{l}^{*}\right)$.

QED

Proof of Proposition 7: *The profit of the high quality firm from selling to both segments in the emerging market exceeds its profit from exclusively selling to the unconstrained segment in the emerging market i.e.* $\Pi_{h2}^{*} > \Pi_{h1}^{*}$*, when consumers' price sensitivity is relatively low, i.e.* $t > t_c$*. In contrast, if consumers' price sensitivity is relatively high, i.e.* $t \le t_c$*, the profit from exclusively selling to the unconstrained segment in the emerging market is relatively higher, i.e.* $\Pi_{h2}^{*} \le \Pi_{h1}^{*}$*. The threshold level of price sensitivity is*

$$t_c = \frac{m - \sqrt{m^2 - n}}{4\lambda(4-\lambda)^2}, \text{ where}$$

$$m = 3(8-5\lambda)^2\left(\chi_c - k_h s_{h2}\right) - 4\lambda(4-\lambda)\left((4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right)$$

$$n = 8\lambda(4-\lambda)^2\left(2\lambda\left((4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right)^2 - (8-5\lambda)^2\left(\chi_c - k_h s_{h2}\right)\left(\beta_{h2} - \beta_l\right)\right)$$

From Appendix 2, the profit of the high quality firm, when it exclusively sells to the unconstrained market segment, is given as follows:

$$\Pi_{h1}^{*} = \frac{\lambda\left(\left(4-\lambda\right)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right)^2}{2t(8-5\lambda)^2}$$

From Appendix 3, the profit of the high quality firm, when it sells to both the unconstrained and cash-constrained market segment, is given as follows: $\Pi_{h2}^{*} = \frac{\left(\chi_c - k_h s_{h2}\right)\left(3t + \beta_{h2} - \alpha_{l2}\right)}{4t}$

The inequality $\left(\Pi_{h1}^{*} > \Pi_{h2}^{*}\right)$ is equivalent to

$$\frac{\lambda\left(\left(4-\lambda\right)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right)^2}{2t(8-5\lambda)^2} > \frac{\left(\chi_c - k_h s_{h2}\right)\left(3t + \beta_{h2} - \alpha_{l2}\right)}{4t}$$

$$2\lambda\left(\left(4-\lambda\right)t + (4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right)^2 > (8-5\lambda)^2\left(\chi_c - k_h s_{h2}\right)\left(3t + \beta_{h2} - \alpha_{l2}\right)$$

$$2\lambda\left(4-\lambda\right)^2 t^2 + 4\lambda\left(4-\lambda\right)\left((4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right)t + 2\lambda(2-\lambda)^2\alpha_{l1}^2$$
$$> (8-5\lambda)^2\left(\chi_c - k_h s_{h2}\right)\left(3t + \beta_{h2} - \alpha_{l2}\right)$$

This is equivalent to the following quadratic inequality in t:

$$2\lambda\left(4-\lambda\right)^2 t^2 + \left[4\lambda\left(4-\lambda\right)\left((4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right) - 3(8-5\lambda)^2\left(\chi_c - k_h s_{h2}\right)\right]t$$
$$+ \left[2\lambda(2-\lambda)^2\alpha_{l1}^2 - (8-5\lambda)^2\left(\chi_c - k_h s_{h2}\right)\left(\beta_{h2} - \alpha_{l2}\right)\right]$$
$$> 0$$

Further simplifying and arranging the terms we can represent $\Pi_{h2}^{*} \le \Pi_{h1}^{*}$ as equivalent to $t \le t_c$, or alternately $\Pi_{h2}^{*} > \Pi_{h1}^{*}$ as equivalent to $t > t_c$, where $t_c = \frac{m - \sqrt{m^2 - n}}{4\lambda(4-\lambda)^2}$,

$$m = 3(8-5\lambda)^2\left(\chi_c - k_h s_{h2}\right) - 4\lambda(4-\lambda)\left((4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right)$$

$$n = 8\lambda(4-\lambda)^2\left(2\lambda\left((4-3\lambda)\alpha_{h1} - (2-\lambda)\alpha_{l1}\right)^2 - (8-5\lambda)^2\left(\chi_c - k_h s_{h2}\right)\left(\beta_{h2} - \beta_l\right)\right)$$

QED

Proof of Proposition 8: *When the high quality firm sells exclusively to the unconstrained segment in the emerging market, the consumer surplus in the emerging market is lower than the consumer surplus in the developed market, i.e.* $CS_1^* < CS_0^*$.

We compare the consumer surplus when the high quality firm H does not sell to a segment of cash-constrained consumers, with the benchmark case where no consumers are cash-constrained. We find that each market segment divides into 3 sub-segments. Let us first consider the unconstrained segment and analyze the change in the consumer surplus of each sub-segment.

1. Consumers located between $\left(0 < x < x_0\right)$ in the unconstrained segment purchase H in the benchmark case and continue to purchase H. They are worse-off since the price of H is higher and its package size is the same, or in other words, the value provided by H decreases.
2. Consumers located between $\left(x_0 < x < x_{u1}\right)$ in the unconstrained segment switch from purchasing L in the benchmark case to purchasing H. We will now show that their consumer surplus decreases.

The change in surplus of consumers in this sub-segment is

$$\begin{aligned}
&\lambda\int_{x_0}^{x_{u1}} U_{h1}(x)dx - \lambda\int_{x_0}^{x_{u1}} U_l(x)dx \\
&= \lambda\int_{x_0}^{x_{u1}} \left(\left(\beta_{h1} - tx\right) - \left(\beta_l - t\left(1-x\right)\right)\right)dx \\
&= \lambda\int_{x_0}^{x_{u1}} \left(\beta_{h1} - \beta_l + t - 2tx\right)dx \\
&= \lambda\left(x_{u1} - x_0\right)\left(\beta_{h1} - \beta_l + t\left(1 - x_{u1} + x_0\right)\right)
\end{aligned}$$

The consumer surplus of this sub-segment decreases iff $\lambda\left(x_{u1} - x_0\right)\left(\beta_{h1} - \beta_l + t\left(1 - x_{u1} + x_0\right)\right) < 0$. Since $\left(x_{u1} - x_0\right) > 0$, we need to prove $\left(\beta_{h1} - \beta_l + t\left(1 - x_{u1} + x_0\right)\right) < 0$. Substituting for β_{h1}, β_l, x_{u1}, x_0 this inequality is equivalent to $\dfrac{(2+\lambda)\alpha_l - (11-8\lambda)\alpha_h - (42-33\lambda)t}{2(8-5\lambda)} < 0$.

Observe that

$$\begin{aligned}
&\frac{(2+\lambda)\alpha_l - (11-8\lambda)\alpha_h - (42-33\lambda)t}{2(8-5\lambda)} \\
&= \frac{(2+\lambda)\left(\alpha_l - \alpha_h\right) - 9(1-\lambda)\alpha_h - (42-33\lambda)t}{2(8-5\lambda)} \\
&= \frac{-(2+\lambda)\left(\alpha_h - \alpha_l\right) - 9(1-\lambda)\alpha_h - (42-33\lambda)t}{2(8-5\lambda)} \\
&< 0
\end{aligned}$$

Thus, the consumer surplus of this sub-segment of unconstrained consumers reduces.

1. Consumers between $\left(x_{u1} < x < 1\right)$ in the unconstrained segment purchase L in the benchmark case and continue to purchase L. They are worse-off since the price of L is higher and its package size is the same, or in other words, the value provided by L decreases.
The above analysis implies that when the high quality firm H does not sell to a segment of cash-constrained consumers, the unconstrained segment is collectively worse-off. Let us now consider the cash-constrained segment and analyze the change in the consumer surplus of each sub-segment.

2. Consumers located between $\left(0 < x < x_{c1}\right)$ in the cash-constrained segment purchase H in the benchmark case, but now do not purchase either product. They are worse-off since they derived positive surplus from purchasing in the benchmark case and now they get no surplus.

3. Consumers located between $\left(x_{c1} < x < x_0\right)$ in the cash-constrained segment switch from purchasing H in the benchmark case to purchasing L. Will now show that their consumer surplus decreases.
The change in surplus of consumers in this sub-segment is

$$
\begin{aligned}
&\left(1-\lambda\right)\int_{x_{c1}}^{x_0} U_{l1}(x)dx - \left(1-\lambda\right)\int_{x_{c1}}^{x_0} U_h(x)dx \\
&= \left(1-\lambda\right)\int_{x_{c1}}^{x_0} \left(\left(\beta_{l1} - t\left(1-x\right)\right) - \left(\beta_h - tx\right)\right)dx \\
&= \left(1-\lambda\right)\int_{x_{c1}}^{x_0} \left(\beta_{l1} - \beta_h - t + 2tx\right)dx \\
&= \left(1-\lambda\right)\left(x_0 - x_{c1}\right)\left(\beta_{l1} - \beta_h - t\left(1 - x_0 + x_{c1}\right)\right)
\end{aligned}
$$

The consumer surplus of this sub-segment decreases iff $\left(1-\lambda\right)\left(x_0 - x_{c1}\right)\left(\beta_{l1} - \beta_h - t\left(1 - x_0 + x_{c1}\right)\right) < 0$

Since $\left(x_0 - x_{c1}\right) > 0$, we need to show that $\left(\beta_{l1} - \beta_h - t\left(1 - x_0 + x_{c1}\right)\right) < 0$. Substituting for β_{l1}, β_h, x_{c1}, x_0 this inequality is equivalent to $\frac{(9\lambda-8)\alpha_h + (8-3\lambda)\alpha_l - (7\lambda+8)t}{2(8-5\lambda)} < 0$

Let $\gamma = \frac{(9\lambda-8)\alpha_h + (8-3\lambda)\alpha_l - (7\lambda+8)t}{2(8-5\lambda)}$. Since $\frac{\alpha_h - \alpha_l}{3} < t < \frac{2\alpha_h + \alpha_l}{6}$, we get $\gamma < \frac{-2\left(\alpha_h - \alpha_l\right)}{6} < 0$. This implies that the consumer surplus of this segment decreases.

Consumers between $\left(x_0 < x < 1\right)$ in the cash-constrained segment purchase L in the benchmark case and continue to purchase L. They are worse-off since the price of L is higher and its package size is the same, or in other words, the value provided by L decreases.

To summarize, when the high quality firm H does not sell to a segment of cash-constrained consumers, the cash-constrained segment is collectively worse-off. The unconstrained segment is also collectively worse-off. Since both segments are worse-off, the aggregate consumer surplus decreases $\left(CS_1^* < CS_0^*\right)$.

QED

Proof of Proposition 9: *When the high-quality firm sells to both segments in the emerging* *market, the consumer surplus in the emerging market is lower than the consumer surplus in the developed market, i.e.* $CS_2^* < CS^*$.

We now compare the consumer surplus when the high quality firm H lowers its price to sell to the cash-constrained segment, with the benchmark case when there are no cash-constrained consumers in the market. In the benchmark case, consumers located between $(0 < x < x_0)$ in both segments purchase H, while those located between $(x_0 < x < 1)$ purchase the low quality firm L. When one segment is cash-constrained, consumers located between $(0 < x < x_2)$ in both segments purchase H, while those located between $(x_2 < x < 1)$ purchase L. Since $x_2 < x_0$, we can compare the purchase behavior of consumers. We find that the market divides into 3 parts:

1. Consumers located between $(0 < x < x_2)$ in both segments continue to purchase from the high quality firm H. They are worse-off since the value provided by H declines.
2. Consumers between $(x_2 < x < x_0)$ in both segments switch from purchasing from the high quality firm H to purchasing from the high quality firm L. We compare their surplus and find that they are also worse-off.
3. Consumers between $(x_0 < x < 1)$ in both segments continue to purchase from the low quality firm L. They are worse-off since L raises its price $(p_{l2} > p_l)$ and keeps the same package size $(s_{l2} = s_l)$, as discussed in Propositions 4 and 5.

Overall, when the high quality firm H lowers its price to serve cash-constrained consumers, the aggregate consumer welfare declines.

QED